Comparative Advantage in Innovation and Production

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This paper develops a dynamic model of innovation and international trade in which agents can direct their research efforts to specific goods in the economy. Trade affects the direction of innovation through its impact on the expected market size for an invention, leading to a two-way relationship between trade and technology absent in standard quantitative Ricardian models. Following a theory-consistent strategy to estimate the extent of endogenous adjustments in technology, I find that they can account for about a half of the observed variance in comparative advantage in production in a sample of 29 countries and 18 manufacturing industries. In addition, the model suggests that standard Ricardian models overestimate the reductions in real income from increases in trade costs, and underestimate the rise in real income due to trade liberalizations.
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In this paper I develop a quantitative model of innovation and trade to study the interactions between trade and technology in a context of endogenous innovation and in which agents can *direct* their research efforts to specific goods in the economy. Ever since the writings of David Ricardo, the relationship between technology and trade has featured prominently in economic analysis. Traditional Ricardian trade theories have emphasized the role of technological differences across countries as the main determinants of specialization patterns in production and trade. The literature following this tradition has typically taken these technological differences as *exogenous* and used static models to analyze topics such as the patterns of production and trade, the welfare gains from trade, the effects of exogenous technological progress and the effects of the diffusion of technology.[1] Moreover, since the seminal contribution of Eaton and Kortum (2002), we have a rich set of *quantitative* trade models incorporating the main Ricardian insights in a context of many goods and many countries, allowing the researcher to go beyond the qualitative analysis that previous models permitted.[2]

Treating technological differences as exogenous, as in a typical static Ricardian model, presumes that the direction of technical change is not affected by trade. However, this is not the case in the presence of endogenous innovation and directed research. Economists have long emphasized the economic nature of innovation activity and the role of

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[1] Grossman and Helpman (1995) survey the literature that follows this tradition.

[2] Eaton and Kortum (2012) survey the quantitative trade literature building on Eaton and Kortum (2002).

expected profits in shaping the amount and direction of innovation efforts. In the words of Schmookler: "*...invention is largely an economic activity, which like other economic activities, is pursued for gain*."[3] Then, in an economy in which research efforts can be *directed* to specific goods, trade can affect innovation and technology through the changes it induces in the expected market size for inventions. These trade-induced changes in market size affect the relative expected profitability of innovation across goods and generate reallocations of research efforts from those goods for which the market contracts towards those goods for which the market expands, ultimately affecting the distribution of technology across goods and countries.

These effects of market size and trade on innovation are not mere theoretical possibilities. Empirical evidence on the effect of market size on innovation goes back to at least the times of Schmookler. In his seminal work on technical change in various capital good industries, *Invention and Economic Growth (1966)*, Schmookler provides evidence about the importance of demand and the expected market size for an innovation as determinants of invention activity. Using changes in demographic trends as a source of exogenous variation in the market size of different types of drugs, Acemoglu and Linn (2004) find economically important effects of these changes in market size on innovation. Recent studies also support the effects of trade on innovation discussed above. Bustos (2011) shows that technology spending increased faster on those Argentinean industries facing higher reductions in Brazil's tariffs in the context of MERCOSUR trade liberalization. Lileeva and Trefler (2010) show that Canadian plants that received preferential access to U.S. markets under the terms of the Canada-U.S. Free Trade Agreement raise their labor productivity by investing in productivity-enhancing activities. This micro-level evidence suggests that current quantitative Ricardian models may be missing an important margin of adjustment in not allowing for trade to affect technology through its impact on the direction of technical change.

The purpose of this paper is to study the two-way relationship between trade and technology that emerges in a context of endogenous innovation and directed research, and to assess qualitatively and quantitatively the macro-level implications of directed research for innovation, production and trade. To that end, I build on Eaton and Kortum (2001) and on recent developments in the static quantitative trade literature to develop a multi-country, general equilibrium, semi-endogenous growth model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of agents to *direct* their research efforts to specific *industries* in a context in which countries differ in their exogenous innovation capabilities; this new element builds into the model the two-way relationship between trade and technology that is the focus of this paper.[4] The semi-endogenous nature of technical change in this model implies that all the effects of directed research are reflected in the levels of the variables of interest in the balanced growth path (BGP), with no effect on BGP-

[3]Schmookler (1966), p. 206.

[4]The fundamental difference with Eaton and Kortum (2001) is not the multiple industry structure per se, but the ability of the economy to direct innovation efforts across these industries. Preserving the undirected nature of innovation in Eaton and Kortum (2001) in a multiple industry model (undirected research within and across industries) would not result in the two-way relationship between trade and technology studied in this paper.

growth rates.[5] These level effects are the focus of this paper. I use the model to shed light on some questions that standard Ricardian quantitative models and reduced-form approaches are not suitable to answer. How important is the feedback from demand and market size to technology? How is comparative advantage in production determined in this context? How are the specialization patterns of innovation, production and trade determined? How does this additional margin affect our conclusions regarding the effects of trade liberalization on production, trade flows and welfare?

Section I and II present the model and the theoretical analysis that guides the quantitative sections. The model features contemporaneous decreasing returns in R&D, which are parsimoniously captured by a single parameter, υ, common to all countries and industries. This parameter can take values on the interval (0, 1), with the lower and upper limits of this interval corresponding to the cases of no innovation and constant returns, respectively. Decreasing returns in R&D control the strength of the endogenous adjustment in technology allowed by directed research, with higher values of υ (weaker decreasing returns) admitting richer interactions between trade and technology. When there are no innovation possibilities, $\upsilon = 0$, the model reduces to the benchmark Ricardian model with exogenous technology. Specifically, the model collapses to a multi-industry version of Eaton and Kortum (2002) (henceforth EK model).[6] Accordingly, under the structure of the model, quantifying the importance of the mechanism proposed in this paper reduces to the determination of the value of the parameter υ.

The model delivers a structural decomposition of comparative advantage in production (CAP) that reflects the main factors affecting the direction of technical change. Specifically, countries' *relative* manufacturing technology in the BGP is a function of exogenous *relative* innovation capabilities and endogenous *relative* market shares, with the importance of the latter controlled by the R&D parameter υ. Relative market shares are determined by relative domestic demand when countries are in autarky, and reflect relative innovation capabilities when trade is frictionless. An implication of this result is that trade can change the pattern of CAP. Moreover, I show that if υ is greater than $1/2$, then adjustments in technology are strong enough to allow for the possibility of home-market effects and reversals in the export profile of countries as trade costs decline.[7]

Directed research also affects the welfare implications of international trade. This point is illustrated by two theoretical results that compare the predictions, conditional on trade and market shares, of the model with directed research with those of the standard Ricardian model with no innovation. First, the Ricardian model overestimates the real income losses from moving to autarky. Second, for the case of two mirror-symmetric countries, the Ricardian model underestimates the real income gains from a uniform reduction in trade costs. A simple intuition lies behind these results. By conditioning on observed market and trade shares, both models imply the same initial levels of manu-

[5]Examples of recent theories of innovation and diffusion of ideas connecting trade and economic growth include Alvarez, Buera and Lucas (2013), Perla, Tonetti and Waugh (2015), Sampson (2015).

[6]Specifically, the model reduces to a multi-industry version of Bernard et al. (2003). However, these models have equivalent reduced-forms at the aggregate level in terms of the determination of wages, production and trade flows.

[7]The existence of home market effects, as defined in Krugman (1980), depends on υ, the size of trade costs, and the size of relative differences in domestic demand. These conditions are discussed in detail in the appendix.

facturing technology and industry price indices. As countries can choose to adjust their manufacturing technology in the innovation model but not in the standard Ricardian model, their real income after the change in trade costs must be higher in the former than in the latter.

The structure of the model facilitates its quantitative implementation. The model in this paper shares the same aggregate cross-sectional structure with a multi-industry EK model with no innovation. An implication of this feature is that many of the methods developed in the literature to estimate the latter can also be applied to the former.[8] Another implication is that both models perform equally well in matching trade and production data in the cross-section. Moreover, the models can be estimated to match exactly the data and to share all exogenous parameters and manufacturing technologies. Nevertheless, even if the two models are set up in this way, they still differ in their counterfactual predictions regarding the changes in trade flows, manufacturing technology and welfare associated with different shocks, all of which are relevant dimensions for policy analysis. Finally, the BGP of the model can be solved in changes by adapting the approach in Dekle, Eaton and Kortum (2008).

Relative to the benchmark Ricardian model with no innovation, the present model adds only one additional parameter to the total number of parameters needed to be estimated to evaluate counterfactual predictions across BGPs. Moreover, armed with an estimate of the parameter υ capturing the decreasing returns in R&D, any method used to estimate the countries' manufacturing productivities in the model with no innovation can be used to recover countries' innovation capabilities. Consequently, all the additional estimation burden imposed by the introduction of directed research relies on the estimation of decreasing returns to R&D.

In section III, I estimate the decreasing-returns parameter υ using production and trade data from 2006 for a sample of 29 countries and 18 manufacturing industries. The estimation strategy is based on the structural equation reflecting the decomposition of CAP discussed above. Due to the two-way relationship between trade and technology in the model, estimating said equation by OLS yields an upward biased estimator of υ. To address this endogeneity problem, I instrument relative market shares with industry preference parameters, under the assumption that the latter are uncorrelated with comparative advantage in innovation. Following this strategy I provide a consistent estimator of υ under the baseline assumption of Cobb-Douglas preferences across industries, an assumption that is pervasive in the quantitative trade literature. I also show that this estimator is biased if the elasticity of substitution across industries, σ, is incorrectly specified. Then, to address concerns of overestimation of υ due to potential upward deviations from the baseline unitary-elasticity assumption, I derive two estimators that provide upper and lower bounds for υ under the weaker assumption that σ is greater than or equal to one. The estimated range of possible values for υ obtained from these estimators, $[0.706, 0.811]$, fits in the $(0, 1)$ interval proposed in the theory and lies completely above the threshold value of $1/2$ discussed above, indicating significant endogenous ad-

[8]In particular, I use the methods developed in Costinot, Donaldson and Komunjer (2012) to estimate comparative advantage in production from trade flows, a variable that is at the center of the analysis of this paper.

justments in technology.

In section IV, I calibrate the model for the same sample of countries and industries used in the estimation of υ. I decompose CAP in the observed open equilibrium in 2006 into exogenous and endogenous components. For the baseline calibration of the decreasing returns in R&D, I find that the endogenous adjustments in technology due to directed research can account for about half of the observed variance in CAP, a share that is robust with respect to the value of the parameter υ in the estimated range of possible values. To assess the importance of *trade-induced* adjustments in technology, I analyze how these results are affected by trade costs. Specifically, I use the model to compute a similar variance decomposition for the cases of frictionless trade and autarky. Moving from autarky to the observed equilibrium in 2006 increases the share of the endogenous component from 26.2% to 52.8%, implying that trade is responsible for about a fourth of the total variance of CAP in the observed equilibrium in 2006. In addition, the share of the endogenous component increases to 94.1% in the frictionless trade equilibrium, suggesting that there is plenty of room for further adjustments in technology as trade frictions decline.

I explore quantitatively two counterfactual situations conditional on observed trade and market shares.

(i) Changes in real income as countries move to autarky. As discussed above, directed research reduces the real-income losses from moving to autarky relative to the standard model with no innovation. However, this effect appears to be quantitatively modest. On average, the real-income losses predicted by the model with directed research represent 93% of those predicted by the model with no innovation. The main reason for these modest differences between the models is the presence of high trade frictions in the observed open equilibrium, which reduces the scope for specialization in innovation.

(ii) 25% reduction in trade costs. The introduction of directed research has relatively important effects on the model's predictions regarding the response of trade flows and market shares. The predicted changes in trade flows in the model with no innovation can explain a little more than a third of the variation in the corresponding changes in the model with directed research. In addition, the model with no innovation tends to underestimate the magnitude of the changes in market shares. According to the model, all countries enjoy an increase in their real income. Consistent with the theoretical result for symmetric countries discussed above, the standard model with no innovation tends to underestimate the rise in real income relative to the model with directed research. However, the differences are also modest in this case. The increases in real income in the presence of directed research are, on average, 2% higher than in the case of no innovation.

As a robustness check, I extend the baseline model to include multiple factors of production, heterogenous trade elasticities across industries and intermediate inputs, and discuss how these extensions may affect the quantitative results above. I argue that many of the these results seem to be more general than what the simple structure of the baseline model would suggest, as some of these extensions affect only the interpretation of some elements of the model. Interestingly, including intermediate goods reduces the estimated value of υ, but has little impact on many of the other results. This is the case because the presence of intermediate goods tends to amplify the overall effect of directed research on manufacturing comparative advantage for a given value of υ, and this overall effect is what the estimations in the baseline model are capturing. In all cases, the main messages of the paper go through, i.e. directed research is an important determinant of comparative advantage in production and trade flows, but it is a somewhat less important factor to understand the effects of trade in manufactured goods on aggregate real income.

This paper contributes to the quantitative trade literature that uses static, multi-industry Ricardian models that build on the EK model, including Arkolakis, Costinot and Rodriguez-Clare (2012), Chor (2010), Caliendo and Parro (2015), Costinot, Donaldson and Komunjer (2012), Levchenko and Zhang (2016) and Shikher (2011). Relative to this literature, in which production technology is exogenous, this paper introduces dynamics, endogenous innovation and directed research into the picture, resulting in an endogenous distribution of production technology across industries that is affected by trade. Interestingly, Kucheryavyy, Lyn and Rodriguez-Clare (2016) show that a multi-industry EK model with Marshallian externalities has some similarities with the BGP of the model in this paper, pointing to another potential driver of endogenous production technology.[9] However, they largely focus on the conditions for existence and uniqueness of the equilibrium, while this paper quantifies the importance of these endogenous adjustments in technology through the structural estimation of the R&D parameter υ.

This paper is related to the large literature estimating increasing returns and home-market effects, such as Davis and Weinstein (1999, 2003), Head and Ries (2001), Feenstra, Markusen and Rose (2001), Hanson and Xiang (2004), and Costinot et al. (2019). After showing that home market effects can arise in the model only when the R&D parameter υ is greater than $1/2$, I structurally estimate this parameter and find that said necessary condition holds in the data. Relative to the literature mentioned above, the main differences are the derivation of home market effects in a Ricardian framework and the structural estimation approach.[10]

This paper is also related to the large endogenous and semi-endogenous growth literature studying the relationship between trade and growth. Among these studies we can mention Grossman and Helpman (1989, 1990, 1991), Rivera-Batiz and Romer (1991), Taylor (1993) and Jones (1995). This paper's main departures from this literature are the introduction of directed research and its quantitative focus. Within this literature, this paper is closest to Eaton and Kortum (2001), which studies the interactions between trade and technology in a single-industry version of the model in this paper. The authors show that trade has no effect on the BGP-level of technology as it brings about two opposing effects that cancel each other out: firms have easier access to foreign markets but also face stronger competition in their domestic markets. In contrast, with multiple industries and directed research, as in the present paper, these two effects of trade do not cancel

[9]Despite these similarities, their model with Marshallian externalities and the one in this paper differ significantly in other dimensions, such as the dynamic response of technology to anticipated shocks. While innovation and technology start to respond as soon as information becomes available (forward looking innovators) in the latter, technology only changes with production when the shock hits in the former (technological change is by-product of production).

[10]This literature has typically followed a reduced-form approach based on extensions of Krugman (1980).

out at the industry level, leading to a reallocation of innovation efforts that is ultimately reflected in the distribution of production technology.

Finally, this paper is also related to Arkolakis et al. (2018). They develop a quantitative single-industry model of multinational production and trade in which firms can separate the location of their innovation and production activities. They use the model to study the effects of openness on countries' specialization patterns between innovation and production, focusing on multinational production as a vehicle through which this international specialization takes place. In contrast, in this paper countries can direct their research efforts to different goods in the economy but cannot separate their innovation and production locations. In this context, I analyze the effects of trade on the innovation and production specialization patterns across the goods in the economy.

I. The Model

In this section I lay out the dynamic model of innovation and trade and characterize the market equilibrium, relegating all derivations to appendix A.1.[11] The distinctive element of the model is the ability of the agents to direct their research efforts to specific goods in the economy, in a context of heterogeneous innovation capabilities across goods and countries. The model is a semi-endogenous growth model, so aggregate growth rates in the BGP are not affected by trade or other standard policies, such as taxes and R&D subsidies. All the effects of directed research on innovation are reflected in the levels of manufacturing technology in the BGP.

A. Basic Environment

Time is continuous and is indexed by $t \in [0, \infty)$. The world consists of N countries. Country i is populated by a continuum of identical and infinitely lived households, each of them with L_{it} members at time t. The mass of households is normalized to one, so L_{it} also represents total population at time t. The representative household in every country grows at the exogenous rate n, i.e., $L_{it} = L_{i0}e^{nt}$. Labor is the only factor of production and its total inelastic supply at time t is given by the population size L_{it}.

There are two sectors in the economy, manufacturing and research. The manufacturing sector produces a fixed set of final goods taking the level of technology as given, while the research sector invests in R&D to improve the technology of final goods.[12] Labor is perfectly mobile across sectors within a country but is immobile across countries.

R&D AND PRODUCTIVITY. — Firms in the research sector invest in R&D to obtain more efficient production techniques for final goods. To capture the idea of directed research, I divide the set of final goods in industries, and allow countries to *direct* their research efforts to any of them. As in Eaton and Kortum (2001), an industry comprises a continuum of goods and research is undirected across them.

[11] The appendix can be found online.

[12] Alternatively, R&D could improve the quality of the product, an equivalent assumption for the analysis.

Formally, a final good in the economy is identified by the pair $(z, \omega) \in [0,1] \times \Omega$, where ω identifies the industry in the fixed set Ω to which the good belongs, and z identifies the good within the industry. There is a fixed set of research firms (of mass one) targeting each industry. Ideas regarding new techniques arrive to individual firms in country i targeting industry ω as a Poisson process with arrival rate $\iota_i^\omega \left(l_t^{R,\omega}\right)^\upsilon$, where ι_i^ω is the exogenous research productivity of country i in that industry, $l_t^{R,\omega}$ is the total number of researchers employed by the representative firm, and $\upsilon \in (0,1)$ is a parameter that captures the extent of contemporaneous decreasing returns in research. An idea is the realization of two random variables, Z and X. The realization of Z indicates the good z within industry ω to which it applies, while the realization of X indicates the efficiency x of the new technique, defined as the number of units of final output that are obtained per unit of labor input, i.e., $q = xl$. Assuming that Z has a uniform distribution over $[0,1]$ and that X has a Pareto distribution with cdf $H(x) = 1 - x^{-\theta}$, the productivity of the best and second best techniques for goods in industry ω available at time t in country i, $\{X_{it}^{\omega,(1)}, X_{it}^{\omega,(2)}\}$, have the following joint distribution,

$$
\begin{aligned}
(1) \qquad F_{it}^{\omega}(x_1, x_2) &= \Pr\left(X_{it}^{\omega,(1)} \le x_1, X_{it}^{\omega,(2)} \le x_2\right) \\
&= \left[1 + T_{it}^{\omega}\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-T_{it}^{\omega} x_2^{-\theta}} \text{ for } x_1 \ge x_2 \ge 1.
\end{aligned}
$$

In the last expression, the scale parameter T_{it}^ω, which I denote the level of manufacturing technology of country i in industry ω, reflects past innovation efforts,

$$
(2) \qquad T_{it}^{\omega} \equiv \iota_i^{\omega} \int_{-\infty}^{t} \left(L_{is}^{R,\omega}\right)^{\upsilon} ds,
$$

where $L_{is}^{R,\omega}$ denotes the total number of researchers in the economy targeting industry ω at time s.[13]

The R&D process described above has three important implications for the purpose of this paper. First, the Inada condition in the expression for the arrival rate of ideas, together with no free entry, guarantees that all countries innovate in all industries in equilibrium.[14] Second, the joint productivity distribution of the best and second best techniques in each industry are as in Bernard et al. (2003) (henceforth BEJK), giving the model a tractable aggregate structure.[15] And third, as becomes clear later, the parameter υ controls the strength of the endogenous adjustment in technology introduced by directed research, with higher values of υ associated with stronger adjustments. As a

[13] T_{it}^ω represents the average number of techniques available for each good (z, ω) at time t in country i.

[14] No free entry in R&D implies that research firms make positive profits in equilibrium. However, the presence of profits per se does not affect the results of the paper, as alternative modeling assumptions with free entry in the research sector and decreasing returns in the mobile factor (labor) yield similar results (e.g. specific research factors).

[15] Throughout the analysis I assume that T_{i0}^ω is sufficiently high for all ω and i, such that we can safely consider that (1) is valid for $x_1 \ge x_2 \ge 0$, as in BEJK. In appendix A.1.1 I show that the difference between F above and a cdf F' given by (1) but with support $x_1 \ge x_2 \ge 0$ becomes negligible as $T_{it}^\omega \to \infty$.

result, the parameter υ is a central element of the analysis.

PREFERENCES. — Household's preferences over streams of per-capita consumption are

$$U_i = \mathbb{E}_0 \left[\int_0^\infty e^{-\bar{\rho} t} L_{it} \frac{(C_{it}/L_{it})^{1-\eta}}{1-\eta} dt \right] = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \frac{C_{it}^{1-\eta}}{1-\eta} dt \right] \tag{3}$$

where η^{-1} is the intertemporal elasticity of substitution, $\rho = \bar{\rho} - n\eta$ is the effective rate of time preference,[16]

$$C_{it} = \exp \left\{ \int_\Omega \alpha_i^\omega \log \left[\frac{C_{it}^\omega}{\alpha_i^\omega} \right] d\omega \right\} \tag{4}$$

and

$$C_{it}^\omega = \left[\int_0^1 c_{it}^\omega (z)^{\frac{\sigma^\omega - 1}{\sigma^\omega}} dz \right]^{\frac{\sigma^\omega}{\sigma^\omega - 1}} \text{ for } \sigma^\omega > 0. \tag{5}$$

FINANCIAL ASSETS AND GEOGRAPHY. — To abstract from issues regarding intertemporal trade and foreign ownership of domestic firms, I assume that financial assets are not traded. This assumption implies that each country must finance all R&D that takes place within its borders with domestic savings, and that trade is balanced every period.[17] However, households in country i can freely borrow and lend at the risk free domestic interest rate r_{it}. Geographic barriers are modeled in the standard iceberg formulation, whereby τ_{ij}^ω units of a good must be shipped from country i in order for 1 unit to arrive to country j, with $\tau_{ij}^\omega > 1$ if $i \neq j$ and $\tau_{ii}^\omega = 1$. Finally, I use w_{it} to denote the wage of country i in period t and set the wage of some country j as the numeraire, $w_{jt} = 1$ for all t.

B. Market Equilibrium

DEMAND. — The representative household in country i maximizes its preferences subject to its budget constraint. Given that preferences are additively separable over time, we can divide the consumer's problem into a static and a dynamic problem. The static problem in each period t involves the optimal allocation of total expenditure E_{it} among the different goods in the economy. The upper tier Cobb-Douglas utility function implies that the share of total expenditure allocated to industry ω in country i is α_i^ω,

$$E_{it}^\omega = \alpha_i^\omega E_{it}. \tag{6}$$

[16] The integral in equation (4) should be understood as a Lebesgue integral, so it can be applied to a set Ω of any cardinality. In the empirical application of the model, the set Ω is finite.

[17] Exogenous trade deficits/surpluses can be introduced without affecting the qualitative results presented in the paper.

Per the CES lower tier utility function, the expenditure on individual goods within that industry is

$$E_{it}^{\omega}(z) = E_{it}^{\omega}\left[p_{it}^{\omega}(z)/P_{it}^{\omega}\right]^{1-\sigma^{\omega}}, \tag{7}$$

where $P_{it}^{\omega} \equiv \left[\int_0^1 p_{it}^{\omega}(z)^{1-\sigma^{\omega}} dz\right]^{\frac{1}{1-\sigma^{\omega}}}$ is the ideal price index of goods in industry ω.

Given the price indices of each industry ω, the aggregate price index is

$$P_{it} = \exp\left\{\int_{\Omega} \alpha_i^{\omega} \log P_{it}^{\omega} d\omega\right\}. \tag{8}$$

From the dual problem of the static problem we get

$$E_{it}^{\omega} = C_{it}^{\omega} P_{it}^{\omega},\ E_{it} = C_{it} P_{it}, \text{ and } E_{it} = \int_{\Omega} E_{it}^{\omega} d\omega. \tag{9}$$

The dynamic problem involves the optimal allocation of expenditure across time subject to an intertemporal budget constraint. The solution to this problem is characterized by the familiar Euler equation

$$\widetilde{C}_{it} = \frac{1}{\eta}\left[r_{it} - \widetilde{P}_{it} - \rho\right], \tag{10}$$

where $\widetilde{X}_t \equiv d\log(X_t)/dt$, together with transversality conditions on bonds holdings.

MANUFACTURING SECTOR. — At any moment in time, there are many alternative techniques in each country to produce a given final good (z, ω) that differ in their respective efficiencies. The owners of these techniques around the world engage in price competition in the market of good (z, ω) in each country i. As a result, the producer with the lowest marginal cost of serving that market becomes the sole supplier, and charges the minimum between the monopoly price and the maximum price that keeps competitors at bay.

The assumptions made so far imply that the demand, market structure and distribution of manufacturing productivities in each industry are as in BEJK, resulting in the same structure of costs, markups and prices. For this reason, when describing the structure of the model within an industry, I describe those aspects of the model that are central to the purpose of this paper, relegating nonessential derivations and proofs to the appendix.

Trade Shares and Prices. Price competition implies that country i buys each good from the cheapest source around the world. The cost distributions I.1 in appendix A.1.2 imply that the fraction of goods from industry ω that country j buys from country i at time t, λ_{ijt}^{ω}, is

$$\lambda_{ijt}^{\omega}=\frac{T_{it}^{\omega}\left(w_{i}\tau_{ij}^{\omega}\right)^{-\theta}}{\Phi_{jt}^{\omega}}. \tag{11}$$

In the previous expression, Φ_{jt}^{ω} is a cost parameter that summarizes how manufacturing technology, wages and trade costs around the world govern the distribution of the cost of serving market ω in country j,

$$\Phi_{jt}^{\omega}=\sum_{k=1}^{N}T_{kt}^{\omega}\left(w_{k}\tau_{kj}^{\omega}\right)^{-\theta}. \tag{12}$$

The cost distributions imply that the distribution of prices in each industry is independent of the source country. Then, (11) also represents the fraction of country j's total expenditure in industry ω that is allocated to goods from country i.

Finally, the exact price index for industry ω in country i is

$$P_{it}^{\omega}=B_{P}^{\omega}\left(\Phi_{it}^{\omega}\right)^{-\frac{1}{\theta}}, \tag{13}$$

where B_{P}^{ω} is a constant that depends only on parameters.[18]

Cost Share in Revenues. As discussed in the appendix, the distribution of costs and markups faced by country j in industry ω imply that production costs represent a fraction $\theta/(1+\theta)$ of its expenditure on that industry. Since said distributions are independent of the source country, $\theta/(1+\theta)$ also represents the share of production costs in the sales of any country i to country j. As this argument is valid for any destination country j and any source country i we get

$$w_{it}L_{it}^{q,\omega}=\frac{\theta}{1+\theta}R_{it}^{\omega}, \tag{14}$$

where $L_{it}^{q,\omega}$ and R_{it}^{ω} are, respectively, the total number of workers employed and the total revenues/sales generated by country i's manufacturing firms in industry ω.

Finally, market clearing at the industry level implies that country i's total manufacturing sales in industry ω, R_{it}^{ω}, equal the world expenditure on industry-ω-goods produced in country i,

$$R_{it}^{\omega}=\sum_{j=1}^{N}\lambda_{ijt}^{\omega}E_{jt}^{\omega}. \tag{15}$$

[18] $B_{P}^{\omega}\equiv\left\{\Gamma\left(\frac{1-\sigma^{\omega}+2\theta}{\theta}\right)\left[1+\bar{m}\left(\sigma^{\omega}\right)^{-\theta}\frac{(\sigma^{\omega}-1)}{[\theta-(\sigma^{\omega}-1)]}\right]\right\}^{\frac{1}{1-\sigma^{\omega}}}$. See appendix A.1.3 for a derivation of (13).

RESEARCH SECTOR. — Firms in the research sector invest in R&D to obtain new production techniques that improve the production efficiency of final goods. Research firms finance their R&D activity issuing equity claims that pay nothing if research efforts fail to improve upon the state of the art technique for some good, but entitle their holders to the stream of future profits if research succeeds. Since no financial assets are traded, the savings of domestic households are the only source of financing for research firms.

Given that there is a continuum $[0, 1]$ of identical firms directing their research efforts to industry ω, and that the risks associated with the R&D efforts are independent across firms, well-diversified equity holders can obtain a deterministic return from their equity investment. Consequently, the equilibrium price of the equity claims issued by the research firms equals their expected return.

Let V_{ijt}^{ω} denote the expected present value at time t of the stochastic future stream of profits generated by an idea from country i in country j, conditional on the idea beating the state of the art in country j at time t. Then

$$V_{ijt}^{\omega} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\int_t^s r_{iu} du} \pi_{js}^{\omega}(z)\, ds \right],$$

where $\pi_{js}^{\omega}(z)$ are the profits at time s of the firm producing good (z, ω) in country j. Notice that the expected profits generated in country j at any future time $s > t$ are equal to average profits at s, multiplied by the probability at time t that the idea is still the state of the art in that country at time s, with this probability given by $\Phi_{jt}^{\omega}/\Phi_{js}^{\omega}$. Using this together with the fact that the share of profits in the sales to country j is $1/(1+\theta)$, the last expression becomes[19]

$$V_{ijt}^{\omega} = \int_t^{\infty} e^{-\int_t^s \left[r_{iu} + \widehat{\Phi}_{ju}^{\omega} \right] du} \frac{E_{js}^{\omega}}{1+\theta} ds. \tag{16}$$

As the last expression shows, in evaluating V_{ijt}^{ω}, future average profits must be discounted at the augmented rate $r_{iu} + \widehat{\Phi}_{ju}^{\omega}$. As more ideas are discovered and the technological frontier in country j grows, some of the firms serving that market at time t are driven out of business by more efficient firms. This endogenous termination probability is captured by the additional term $\widehat{\Phi}_{ju}^{\omega}$ in the discount rate.

Given that the price of an equity claim issued by a research firm equals its expected value, any research firm maximizes the expected returns from the R&D activity. On the one hand, for any firm in country i using $l_t^{R,\omega}$ researchers to target industry ω, the expected benefit over the interval dt is $\frac{\iota_i^{\omega}\left(l_t^{R,\omega}\right)^{\upsilon}}{T_{it}^{\omega}} \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{ijt}^{\omega} \right] dt$, where $\iota_i^{\omega}\left(l_{it}^{R,\omega}\right)^{\upsilon} dt$ is the probability of having and idea in the interval dt, $1/T_{it}^{\omega}$ is the probability that the idea beats the technological frontier in country i, and λ_{ijt}^{ω} is the probability of beating the state of the art technique in country j conditional on beating the frontier in country i. On

[19] From the analysis of the costs share in revenues we know that the total production costs in country i's sales to country j is given by $\theta/(1+\theta)$, which implies that the share of profits is $1/(1+\theta)$.

the other hand, the costs for the firm are simply the wages paid to researchers $w_t l_t^{R,\omega}$. The first-order condition of this problem yields the following aggregate equilibrium condition

$$w_{it} = \frac{\iota_i^{\omega} \upsilon \left(L_{it}^{R,\omega}\right)^{\upsilon-1}}{T_{it}^{\omega}} \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{ijt}^{\omega}\right], \tag{17}$$

where $L_{it}^{R,\omega}$ is the total number of researchers targeting industry ω.

It is worth mentioning that, in this setting, research firms make positive profits in equilibrium. However, the presence of these profits does not affect the households' saving decisions nor the direction of innovation efforts in the economy.

BALANCED TRADE AND LABOR MARKET CLEARING. — Since there is no trade in financial assets, households in country i can save only in equity claims issued by the domestic research sector. Consequently, at any moment in time, total income is equal to the sum of total expenditure in final goods and total purchases of domestic equity claims

$$R_{it} + \int_{\Omega} \left[\Pi_{it}^{R,\omega} + w_{it} L_{it}^{R,\omega}\right] d\omega = \int_{\Omega} I_{it}^{\omega} d\omega + E_{it},$$

where $R_{it} \equiv \int_{\Omega} R_{it}^{\omega} d\omega$ is the total revenue generated by manufacturing firms,[20] $\Pi_{it}^{R,\omega}$ are the total profits generated by the research firms in industry ω, and I_{it}^{ω} is the total value of household's purchases of equity claims issued by research firms in industry ω. By definition, the total revenue of research firms in industry ω equals I_{it}^{ω}, so the last expression implies that trade is balanced every period,[21]

$$R_{it} = E_{it}. \tag{18}$$

The labor market clearing condition requires that the sum of the total number of production workers and researchers allocated to all industries equals the total endowment of labor at each moment in time,

$$L_{it} = \int_{\Omega} \left[L_{it}^{q,\omega} + L_{it}^{R,\omega}\right] d\omega. \tag{19}$$

Conditions (2), (7)-(19) fully describe the equilibrium, prompting the formal definition in the appendix. In the rest of the paper, I use L_{it}^{ω}, L_{it}^{q} and L_{it}^{R} to denote the total number of workers employed in industry ω, in the manufacturing sector and in the research sector, respectively, i.e. $L_{it}^{\omega} = L_{it}^{q,\omega} + L_{it}^{R,\omega}$, $L_{it}^{q} = \int_{\Omega} L_{it}^{q,\omega} d\omega$, and $L_{it}^{R} = \int_{\Omega} L_{it}^{R,\omega} d\omega$.

[20] Recall that $R_{it}^{\omega} \equiv \Pi_{it}^{q,\omega} + w_{it} L_{it}^{q,\omega}$, where $\Pi_{it}^{q,\omega}$ are total profits generated by manufacturing firms in class ω.

[21] Trade is not required to be balanced in each industry ω, implying that in general $R_{it}^{\omega} \neq E_{it}^{\omega}$.

II. Balanced Growth Path

In this section I will focus on the BGP of the economy in which $\{P_{it},\ R_{it},\ E_{it},\ C_{it},\ C_{it}^{\omega},\ P_{it}^{\omega},\ R_{it}^{\omega},\ E_{it}^{\omega},\ V_{it}^{\omega},\ L_{it}^{q,\omega},\ L_{it}^{R,\omega},\ T_{it}^{\omega}\}$ grow at constant rates for every country i and industry ω. As a general rule, throughout the rest of the paper I omit the subscript t when referring to the BGP-level of variables that are constant in the BGP. Finally, all derivations are relegated to appendix A.2.

A. Solving for the BGP

Growth rates in the BGP depend only on the exogenous rate of population growth and technological parameters, so trade does no affect them. Instead, the effects of trade on the direction of innovation efforts are reflected in the BGP-*levels* of the variables of interest, such as technology, consumption, price level, etc. In this section, I focus on these level-effects and relegate the analysis of growth rates to Lemma A.2 in appendix A.2.1.

DIRECTED RESEARCH AND ENDOGENOUS MANUFACTURING TECHNOLOGY. — A factor affecting the expected benefits of R&D is the expected size of the market for a future innovation. For this reason, a key variable in the study of the determinants of country i's BGP-levels of technology is its market share in industry ω, $\beta_i^{R,\omega}$, defined as the ratio of country i's total sales in industry ω, R_{it}^{ω}, to the world expenditure in the same industry, $E_t^{\omega} \equiv \sum_{i=1}^{N} E_{it}^{\omega}$, i.e., $\beta_i^{R,\omega} \equiv R_{it}^{\omega}/E_t^{\omega}$. As shown later, $\beta_i^{R,\omega}$ is constant in the BGP, so it also represents the fraction of present and future profits generated by world expenditure on industry ω accruing to firms in country i.

Combining Lemma A.2.iii, equation (17) and the definitions of $\widetilde{T}_{it}^{\omega}$ and $\beta_i^{R,\omega}$ we obtain the following equilibrium relationship in the BGP,

$$T_{it}^{\omega} = B_T \iota_i^{\omega} \left[\beta_i^{R,\omega} V_t^{\omega}/w_i\right]^{\upsilon}, \tag{20}$$

where $B_T \equiv n^{\upsilon-1}\upsilon^{2\upsilon-1}$ is a constant and V_t^{ω} is the present value of the profits generated by the stream of world expenditure $\left\{E_s^{\omega}\right\}_{s\geq t}$ in the industry. The last equation identifies the research productivity ι_i^{ω}, the market size captured by country i's firms $\beta_i^{R,\omega} V_t^{\omega}$, and the cost of researchers w_i, as the only aggregate channels through which all exogenous parameters of the model can affect manufacturing productivity.[22] In particular, factors such as openness, comparative advantage in innovation, country size and home market effects, all affect T_{it}^{ω} through their effect on $\beta_i^{R,\omega} V_t^{\omega}$ and w_i.[23]

To understand the effects of trade on technology in the presence of directed research it is instructive to compare the present model with the single industry case in which, by

[22] No closed form solutions exist for T_{it}^{ω} in terms of primitives except for the autarky case.

[23] The relationship between T_{it}^{ω} and ι_i^{ω} in (20) captures only the direct effect of ι_i^{ω} on T_{it}^{ω}, without considering the potential indirect effects through $\beta_i^{R,\omega} V_t^{\omega}/w_i$.

construction, no directed research takes place. I do this with the help of the equation (21) that relates industry manufacturing technology to the resources allocated to the industry. Letting $\delta_i^\omega \equiv L_{it}^\omega / L_{it}$ denote the share of the total labor force allocated to industry ω, the definition of $\widehat{T}_{it}^\omega$ and points iii and iv of Lemma A.2 yield

$$T_{it}^\omega = B_T' \iota_i^\omega \left[\delta_i^\omega L_{it}\right]^\upsilon , \tag{21}$$

where $B_T' \equiv (\upsilon n)^{-1} \kappa^\upsilon$ is a constant.

For the single-industry case, Eaton and Kortum (2001) show that the degree of openness has no effect on the BGP-level of technology. This can be seen in equation (21) noting that in this case all the resources must be allocated to the sole industry in the economy regardless of the degree of openness, i.e., $\delta_i^\omega = 1$. In this case, openness brings about two opposing effects on innovation that cancel each other out: on the one hand, there is a positive effect on innovation granted by the easier access to foreign markets experienced by domestic firms; on the other hand, there is a negative effect on innovation caused by the increased competition faced by those same firms in their domestic markets. Then, in the single-industry case, the BGP-level of technology of a country depends only on its research productivity and on the size of its labor force, $T_{it}^\omega = B_T' \iota_i^\omega L_{it}^\upsilon$, and it is not affected by trade.

In contrast, in the multi-industry case, when a country opens up to trade it reallocates its research efforts towards those industries that present more profitable investment opportunities. As a result, the two opposing effects on innovation described above do not cancel out at the industry level, so openness has an effect on the distribution of manufacturing technology in the BGP. This is captured in equation (21) by the endogenous trade-induced adjustments of industry labor-shares δ_i^ω.

The effects of specialization and trade on the allocation of resources across industries in the BGP can be seen with the help of the next equation

$$\delta_i^\omega = \frac{\beta_i^{R,\omega}}{\beta_i^R} \alpha^\omega = \frac{\beta_i^{R,\omega}}{\beta_i^{E,\omega}} \alpha_i^\omega , \tag{22}$$

where the variable $\alpha^\omega \equiv E_t^\omega / E_t$ is the share of world expenditure allocated to industry ω, $\beta_i^R \equiv R_{it}/R_t$ is the share of country i in world output, and $\beta_i^{E,\omega} \equiv E_{it}^\omega / E_t^\omega$ is the share of country i's expenditure in world expenditure in industry ω.[24]

The first equality in (22) shows the effects of specialization and world demand on manufacturing technology. On the one hand, the ratio $\beta_i^{R,\omega}/\beta_i^R$ can be interpreted as a measure of specialization: a value of this ratio above one means that country i contributes more to world output in industry ω than to total world output, reflecting a specialization in production and R&D due to more primitive supply and demand factors. On the other hand, α^ω reflects the effects of world demand on δ_i^ω: a greater world demand for goods in industry ω—as captured by α^ω—is associated with more production and R&D in that

[24] R_{it} denotes country i's total output, $R_{it} \equiv \int_\Omega R_{it}^\omega d\omega$; and R_t denotes world output, $R_t \equiv \sum_{i=1}^N R_{it}$.

industry in every country, leading to a worldwide higher BGP-level of the manufacturing technology in that industry.

The second equality in (22) illustrates the effects of trade on δ_i^ω and T_{it}^ω. In autarky, a country must consume what it produces, so production must equal expenditure in every industry, i.e. $\beta_i^{R,\omega}/\beta_i^{E,\omega} = 1$ for all ω. In this case, the allocation of resources across industries is driven only by demand conditions in country i as captured by α_i^ω. In contrast, when a country trades with the world, trade does not need to be balanced in each industry and the ratio $\beta_i^{R,\omega}/\beta_i^{E,\omega}$ can differ from one. If $\beta_i^{R,\omega}/\beta_i^{E,\omega} > 1$ (< 1), then country i is a net exporter (importer) in industry ω and its level of technology is higher (lower) in that industry relative to an autarkic economy that shares the same fundamental parameters.

Finally, equations (20) and (21) show how the parameter υ controls the importance of the endogenous adjustments of technology: the weaker the decreasing returns to R&D (high υ), the stronger the endogenous effects of directed research on technology.

EXISTENCE OF A BALANCED GROWTH PATH. — The next proposition gathers all the equations from which the BGP of this economy is obtained and guarantees the existence of a solution. Although the system can be simplified even further, the system of equations presented in the proposition helps identify the new elements of the model and facilitates the comparison with a benchmark model with no directed research.

PROPOSITION 1: *The BGP-values of trade shares λ_{ij}^ω, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, world-wide industry expenditure shares α^ω, countries' levels of manufacturing technology T_{it}^ω and wages w_i are obtained as a solution to the following system of equations*

(23)

$$\lambda_{ij}^\omega = \frac{T_{it}^\omega\left(w_i\tau_{ij}^\omega\right)^{-\theta}}{\sum_{k=1}^N T_{kt}^\omega\left(w_k\tau_{kj}^\omega\right)^{-\theta}} \quad (23.1) \qquad \beta_i^R = \int_\Omega \alpha^\omega\beta_i^{R,\omega}d\omega \quad (23.5)$$

$$\beta_i^{R,\omega} = \sum_{j=1}^N \lambda_{ij}^\omega\beta_j^{E,\omega} \quad (23.2) \qquad \alpha^\omega = \sum_{j=1}^N \alpha_j^\omega\beta_j^E \quad (23.6)$$

$$\beta_i^E = \beta_i^R \quad (23.3) \qquad \beta_i^{E,\omega} = \frac{\alpha_i^\omega\beta_i^E}{\alpha^\omega} \quad (23.7)$$

$$T_{it}^\omega = B_T'\iota_i^\omega\left[\frac{\beta_i^{R,\omega}}{\beta_i^R}\alpha^\omega L_{it}\right]^\upsilon \quad (23.4) \qquad \beta_i^R = \frac{w_i L_{it}}{\sum_{j=1}^N w_i L_{it}} \quad (23.8)$$

for all i,ω. Moreover, for $\upsilon \in (0,1)$ a solution to the system exists with $\beta_i^{R,\omega} > 0$ for all i,ω.[25]

[25]Kucheryavyy, Lyn and Rodriguez-Clare (2016) independently derive conditions for existence and uniqueness of a solution to a system of equations similar to the one presented here.

Proof. See appendix A.2.5.

The equations in the first column of the system (23) represent structural equations of the model stated previously in the text but now expressed in terms of shares: the first line reproduces the expression for trade shares obtained in (11); the second line is the market clearing condition for the output of each country in each industry (equation (15)); the third line is the balanced trade condition (18); and the fourth line combines the technology relationships (21) and (22). The equations in the second column express all the relationships between expenditure shares, $\{\beta_i^{E,\omega}, \beta_i^E\}$, market shares $\{\beta_i^{R,\omega}, \beta_i^R\}$, industries' shares in world expenditure, α^ω, and wages, w_i, that follow immediately from their definitions.

With the exception of equation (23.4), the rest of the system (23) contains the exact same equilibrium equations corresponding to a multi-industry benchmark model with no endogenous innovation, in which the distribution of firms' productivities within an industry is given by (1) for some exogenous scale parameter T_{it}^ω, and in which the structure of the manufacturing sector is the same as in the present model.[26] Moreover, this benchmark model shares the same equilibrium equations with a Ricardian multi-industry EK model with no innovation and a common θ across industries.[27] Relative to that benchmark, Proposition 1 shows that the effect of directed research on the BGP-levels of manufacturing technology is completely captured by equation (23.4).

The observations in the last paragraph have the following implications. First, the parameter υ, that captures the decreasing returns in R&D, controls the relevance of the effects of directed research on manufacturing technology. In particular, when this parameter is set to zero, all the endogenous adjustments of technology are eliminated and we are back to the no innovation benchmark model for some given initial levels of technology. In this sense, the present model nests the benchmark model with no innovation. Second, relative to the model with no innovation, the introduction of directed research adds only one parameter to the total number of parameters relevant to the BGP of the model. Moreover, armed with an estimate of υ, any method used to estimate the manufacturing technologies T_{it}^ω in the model with no innovation can be used to recover the underlying research productivities ι_i^ω through equation (23.4).[28] Consequently, all the additional estimation burden imposed by the present model is related to the estimation of υ.

Third, the fact that the model with and without innovation share the same cross-sectional structure imply that both models perform equally well in matching trade, production and consumption data in the cross-section. Moreover, if we allow for exogenous trade deficits in (23.3), the two models can be estimated to match exactly the aforementioned data and to share all exogenous parameters and T_{it}^ω. Nevertheless, even if the two models are set up in this way, they still differ in their counterfactual predictions regarding the changes in trade flows, manufacturing technology and welfare associated with

[26]This benchmark model is a multi-industry BEJK model with a common shape parameter θ across industries.

[27]Some studies using models with this structure include Costinot, Donaldson and Komunjer (2012), Chor (2010) and Shikher (2011), among others.

[28]Specifically, the estimation of T_{it}^ω in the BGP of this model is compatible with any method that does not assume that R&D and manufacturing technology are not affected by trade flows.

different shocks, all of which are relevant dimensions in policy analysis.

THE BGP IN CHANGES. — To perform counterfactual analysis across BGPs, I solve the system (23) in changes, extending the approach popularized by Dekle, Eaton and Kortum (2008). A well-known advantage of this approach is that only a subset of parameters is needed for counterfactual evaluations. In addition, armed with an estimate of the parameter capturing the decreasing returns in R&D, performing counterfactual analysis in the model with directed research does not impose any additional data requirement over a multi-industry EK model with no innovation. The details of this approach are outlined in appendix A.2.8.

B. Trade and Comparative Advantage in Production

In this section, I study the determinants of comparative advantage in production (CAP) in the BGP, emphasizing the role of the decreasing returns in R&D in controlling the extent of its endogenous adjustments. This section provides the theoretical foundation for the estimation strategy of the parameter υ.

I start by defining the concepts of comparative advantage used in the analysis. For any pair of countries $\{i, i'\}$ and any pair of industries $\{\omega,\omega'\}$, country i has CAP in industry ω in period t if $T^{\omega}_{it}/T^{\omega'}_{it} > T^{\omega}_{i't}/T^{\omega'}_{i't}$.[29] Similarly, country i has comparative advantage in innovation in industry ω if $\iota^{\omega}_{i}/\iota^{\omega'}_{i} > \iota^{\omega}_{i'}/\iota^{\omega'}_{i'}$. For this reason, I refer to the distribution of the double ratios $\left(T^{\omega}_{it}/T^{\omega'}_{it}\right) / \left(T^{\omega}_{i't}/T^{\omega'}_{i't}\right)$ and $\left(\iota^{\omega}_{i}/\iota^{\omega'}_{i}\right) / \left(\iota^{\omega}_{i'}/\iota^{\omega'}_{i'}\right)$ as comparative advantage in production and innovation, respectively.

Taking double ratios in equation (20) for any pair of countries $\{i, i'\}$ and any pair of industries $\{\omega, \omega'\}$, we can express CAP as the product of an exogenous component given by comparative advantage in innovation, and an endogenous component that captures the effects on technology of differences in relative specialization in R&D,

$$\underbrace{\frac{T^{\omega}_{it}/T^{\omega}_{i't}}{T^{\omega'}_{it}/T^{\omega'}_{i't}}}_{\substack{\text{Comparative Adv.}\\ \text{in Production (CAP)}}} = \underbrace{\frac{\iota^{\omega}_{i}/\iota^{\omega}_{i'}}{\iota^{\omega'}_{i}/\iota^{\omega'}_{i'}}}_{\substack{\text{Exogenous}\\ \text{Comparative Adv.}\\ \text{in Innovation}}} \times \underbrace{\left(\frac{\beta^{R,\omega}_{i}/\beta^{R,\omega}_{i'}}{\beta^{R,\omega'}_{i}/\beta^{R,\omega'}_{i'}}\right)^{\upsilon}}_{\substack{\text{Endogenous}\\ \text{Component}}}. \tag{24}$$

The last expression shows that the parameter υ also represents the elasticity of CAP with respect to R&D specialization. Then, the importance of the endogenous component in the last decomposition depends negatively on the strengths of the decreasing returns in innovation—positively on υ—, with the extreme case of $\upsilon = 0$ leading to an exogenous distribution of CAP.

[29] The levels of technology are given in period t, so they determine the autarky industry price indices in that period.

The complexity of the interactions among innovation, production and prices precludes any analytic characterization of the endogenous component in (24) in terms of exogenous parameters for the general case of Proposition 1. However, there are two special cases in which such a characterization can be obtained: the case of frictionless trade (zero gravity), in which $\tau_{ij}^{\omega} = 1$ for all i, j, ω; and the case in which geographic barriers are prohibitive (autarky), with $\tau_{ij}^{\omega} \to \infty$ for $j \neq i$.[30] The characterization of CAP in these two extreme cases is presented in the following lemma.

LEMMA 1: *Let the subscripts a and zg denote autarky and zero gravity, respectively. For any pair of countries* $\{i, i'\}$ *and any pair of industries* $\{\omega, \omega'\}$, *CAP is given by*
(i) in autarky

$$\frac{T_{it,a}^{\omega}/T_{it,a}^{\omega'}}{T_{i't,a}^{\omega}/T_{i't,a}^{\omega'}} = \frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}} \left[\frac{\alpha_i^{\omega}/\alpha_i^{\omega'}}{\alpha_{i'}^{\omega}/\alpha_{i'}^{\omega'}} \right]^{\upsilon}, \tag{25}$$

(ii) in a zero gravity world

$$\frac{T_{it,zg}^{\omega}/T_{it,zg}^{\omega'}}{T_{i't,zg}^{\omega}/T_{i't,zg}^{\omega'}} = \frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}} \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}} \right]^{\frac{\upsilon}{1-\upsilon}}. \tag{26}$$

Proof. See appendix A.2.6.

Equation (25) shows that in autarky, the endogenous component of CAP depends on countries' relative expenditure shares across industries. Given that in autarky countries must produce what they consume, autarkic economies innovate more and produce more in those industries in which their domestic demand is higher. The effect of this demand-induced specialization in innovation on CAP is captured in the term in brackets in (25). In contrast, in a zero gravity world, the *relative* specialization patterns of innovation and production are no longer affected by domestic demand conditions. Instead, they reflect fundamental differences in innovation capabilities across industries as captured by comparative advantage in innovation. When countries open up to trade, they direct their research efforts towards those industries in which they have comparative advantage in innovation. Over time, as the result of innovation efforts translates into more efficient techniques, manufacturing technology starts to reflect the underlying specialization in innovation, which ultimately leads to the distribution of CAP in Lemma 1.ii.

Lemma 1 has the following implications. First, as trade costs fall and countries become more integrated, the observed dispersion in the profile of CAP could rise or fall depending on the relative dispersions of relative local demand conditions and compara-

[30]Although the case of autarky allows for a full analytic solution of the model, this is not true in the case of zero gravity, since there is no closed form solution for relative wages. However, the structure of the model implies that relative wages do not affect CAP, allowing for a closed-form characterization.

tive advantage in innovation.[31] This observation is especially relevant to the evaluation of the gains from trade, as relative technological differences are the source of those gains in the standard Ricardian model (corresponding to $\upsilon = 0$). Then, assessing the effects on the gains from trade of observed changes in CAP through the lenses of that model may lead to incorrect conclusions if those changes are themselves the consequence of endogenous changes in innovation induced by changes in the trade environment.

Second, reductions in trade costs could potentially reverse the profile of CAP across countries if the difference between relative domestic demand conditions and relative innovation capabilities across industries is sufficiently large. Moreover, if the contemporaneous decreasing returns to R&D are not too strong, reductions in trade costs can even generate a reversal in the export profile of countries. To see this, consider the effects on the trade balance as trade costs increase from a zero gravity world to autarky in the following context: for some pair of countries $\{i,i'\}$ and some pair of industries $\{\omega,\omega'\}$, country i has comparative advantage in innovation in industry ω' and a relatively stronger demand for goods in industry ω. When trade is frictionless, country i's relatively strong demand for goods in industry ω, together with its CAP in industry ω', implies that the country is a net importer in industry ω.[32] As transport costs increase, countries reallocate their production and innovation efforts towards those goods in which they have a strong domestic demand. In the example given, the effects of these reallocations on technology and production improve the trade balance of country i in industry ω. If the decreasing returns to R&D are sufficiently weak –high υ–, then the adjustment on manufacturing technology can be strong enough to make country i a net exporter in industry ω. The next lemma formalizes this argument for the case of two mirror symmetric countries and two industries.

LEMMA 2: *Consider an economy with two mirror symmetric countries $i = 1, 2$, two industries $\{\omega, \omega'\}$, and symmetric uniform trade costs across industries, i.e. $\tau^{\omega}_{ij} = \tau$ for all ω, i, j such that $i \neq j$.*[33] *Assume that country 1 has a relative strong demand for goods in industry ω and a comparative advantage in innovation in industry ω' such that the following condition holds*

$$\frac{\alpha^{\omega}_1/\alpha^{\omega}_2}{\alpha^{\omega'}_1/\alpha^{\omega'}_2} > 1 > \frac{\iota^{\omega}_1/\iota^{\omega}_2}{\iota^{\omega'}_1/\iota^{\omega'}_2}. \tag{27}$$

Then, the countries will display a reversal in their export profile as they move from au-

[31] In particular, using log standard deviation as a measure of dispersion we get

$$sd\left(\log\left(\frac{T^{\omega}_{it,a}/T^{\omega'}_{it,a}}{T^{\omega}_{i't,a}/T^{\omega'}_{i't,a}}\right)\right) \geq sd\left(\log\left(\frac{T^{\omega}_{it,zg}/T^{\omega'}_{it,zg}}{T^{\omega}_{i't,zg}/T^{\omega'}_{i't,zg}}\right)\right) \Longleftrightarrow sd\left(\log\left(\frac{\alpha^{\omega}_{i}/\alpha^{\omega'}_{i}}{\alpha^{\omega}_{i'}/\alpha^{\omega'}_{i'}}\right)\right) \geq \frac{1}{1-\upsilon} sd\left(\log\left(\frac{\iota^{\omega}_{i}/\iota^{\omega'}_{i}}{\iota^{\omega}_{i'}/\iota^{\omega'}_{i'}}\right)\right).$$

[32] Because of balanced trade, this implies that country i is net exporter of goods in industry ω' in a zero gravity world.

[33] The countries are mirror images of each other. See the proof of the Lemma in the appendix for a precise definition.

tarky to frictionless trade if, and only if,

$$\left(\frac{\alpha_1^\omega}{\alpha_2^\omega}\right)^{\upsilon-\frac{1}{2}} > \frac{\iota_2^\omega}{\iota_1^\omega}. \tag{28}$$

Proof. See Appendix A.2.7.

The possibility of reversals in the export profile of countries is closely connected to the presence of home market effects in the model. In particular, condition (28) implies that a reversal in the export profile of countries never arises if $\upsilon \in [0, 1/2)$, which is the same range of values of υ for which the model does not exhibit home market effects.[34] In this sense, Lemma 2 provides a theoretical threshold for the parameter υ above which the endogenous adjustments in technology are strong enough to allow for the possibility of home market effects and potential reversals in the export profile of countries.

Finally, Lemma 1 implies that in any trading equilibrium that is far from the extreme cases of autarky and zero gravity, the endogenous component of CAP should be correlated with both, comparative advantage in innovation and relative domestic demand. This observation plays an important role in the estimation of parameter υ.

C. Real Income in the BGP

Country i's real income per-capita at any time t, W_{it}, is given by

$$W_{it} = B_P \exp\left\{\int_\Omega \log\left(T_{it}^\omega\right)^{\alpha_i^\omega/\theta} d\omega\right\} \exp\left\{\int_\Omega \log\left(\lambda_{ii}^\omega\right)^{-\alpha_i^\omega/\theta} d\omega\right\}. \tag{29}$$

The last expression is valid for all values of the R&D parameter υ, including $\upsilon = 0$, corresponding to the model with no innovation. However, when $\upsilon > 0$ technology levels endogenously adjust in response to a shock, providing an extra channel through which real income is affected.

Consider the case of a foreign shock, defined as a change in foreign research productivities, foreign labor endowments or trade costs that do not affect country i's ability to serve its domestic market. Letting $\widehat{X} \equiv X'/X$ denote the relative change in variable X, the impact of such a shock on the BGP-level of country i's real income per-capita can be computed as

$$\widehat{W}_{it} = \exp\left\{\upsilon \int_\Omega \log\left(\widehat{\delta}_i^\omega\right)^{\alpha_i^\omega/\theta} d\omega\right\} \exp\left\{\int_\Omega \log\left(\widehat{\lambda}_{ii}^\omega\right)^{-\alpha_i^\omega/\theta} d\omega\right\}. \tag{30}$$

The last expression offers a parsimonious way to evaluate *ex-post* the change in real income associated with a foreign shock. Armed with estimates of the preference para-

[34] See appendix A.2.10 for a formal discussion of the home market effect in this model.

meters α_i^ω, the R&D parameter υ, and the dispersion parameter θ, the ex-post evaluation of (30) only requires information on the change in the home shares of expenditure, $\widehat{\lambda}_{ii}^\omega$, and the change in the industry shares of total output, $\widehat{\delta}_i^\omega$. Relative to the model with no innovation, the formula for $\widehat{W}_{it}$ must be augmented by the first term in (30) to capture the endogenous adjustments in technology.[35] Not accounting for these technology adjustments in the *ex-post* evaluation of $\widehat{W}_{it}$ may result in a higher or lower value, as the contribution of the new term cannot be determined at this level of generality.

Let us now turn to the analysis of the counterfactual predictions of the model regarding the effect of changes in trade costs on real income per-capita in the BGP. To evaluate the change in real income according to (30), we first need to use the model to evaluate the changes in the home share of expenditures, $\widehat{\lambda}_{ii}^\omega$, and the share of each industry in total output, $\widehat{\delta}_i^\omega$, associated with the change in trade costs. Throughout the analysis this is done as follows: (i) I calibrate the system (23) in changes (system A.8 in the appendix) to a baseline equilibrium using information on endogenous trade shares λ_{ij}^ω, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, world-wide industries' expenditure shares α^ω and the parameter θ; (ii) I solve the system in changes for some value of the parameter υ; and (iii) I compute the change in real income per capita according to (30). In this way, the differences between the changes in real income per capita predicted by the models with and without innovation are those that emerge from setting $\upsilon > 0$ or $\upsilon = 0$ in the system in changes. Note that in general, said differences not only reflect the extra term in (30), but also the models' different predictions for the changes in trade flows. Proposition 2 summarizes the effects of directed research on the predicted changes in real income associated with a change in trade costs.

PROPOSITION 2: *(i) Consider a world economy of two mirror symmetric countries. Starting from an initial open economy equilibrium, a uniform decrease (increase) in trade costs generates a larger increase (lower reduction) in the BGP-level of real income per capita in the model with directed research than in the model with no innovation.*
(ii) For the general asymmetric case, moving to autarky generates lower reductions in the BGP-level of real income per capita in the model with directed research than in the model with no innovation.

Proof. See Appendix A.2.9.

Proposition 2 compares the predictions of the models with and without innovation regarding the changes in real income per-capita *conditional* on observed trade shares and market shares in the original equilibrium. In this sense, this comparison is consistent with the *ex-ante* analysis in Arkolakis, Costinot and Rodriguez-Clare (2012). However, Proposition 1 implies that the models with and without innovation can be calibrated to share all exogenous parameters (other than υ) and manufacturing technology in the initial equilibrium. Consequently, the comparison in Proposition 2 is also compatible

[35]The formula for the model with no innovation is given by the second term in (30) and it is obtained by setting $\upsilon = 0$ in that expression. In this case, the formula reduces to the expression found in Arkolakis, Costinot and Rodriguez-Clare (2012) for the case of the multi-industry Eaton and Kortum (2002) model.

with the theoretical comparative static exercises in Melitz and Redding (2015).[36] In this way, the changes in real income in the model with no innovation can be interpreted as the changes that arise in the model with innovation when technology is not allowed to adjust. Under this interpretation, the results in Proposition 2 are very intuitive. Directed research introduces a new margin through which economies can adjust to the change in trade costs. Then, a simple revealed preference argument implies that in the model with directed research, economies can enjoy a higher level of real income after the change in trade costs, regardless of the direction of the change. The proof in the appendix formalizes this argument.

III. Estimating the Decreasing Returns in R&D

The decreasing-returns parameter υ plays a central role in the model as it controls the strengths of the endogenous adjustment in technology. In this section, I estimate this key parameter from production and bilateral trade data, following a theory-consistent estimation strategy based on the structural decomposition of comparative advantage given in equation (24) and on the insights from Lemma 1.

A. Data

I identify the industries in the model with manufacturing industries corresponding roughly to two-digit ISIC Rev.3 classification, giving a total of $\Omega = 18$ industries. The data on trade flows is obtained from the OECD STAN (Structural Analysis) Database, while production data is sourced form the 2012 UNIDO Industrial Statistic Database INDSTAT2. The sample of countries include 25 OECD countries, 4 non-OECD countries and a constructed rest of the world aggregate, yielding a sample of $N = 29$. I focus mostly on OECD countries as I consider the model to be a better representation of relatively advanced economies.

I map the variables in the model to the data as follows. (i) Manufacturing firms' total revenue by industry, R^{ω}_{it}, is given by gross production; (ii) exports X^{ω}_{it} and imports M^{ω}_{it} are obtained directly from bilateral trade data; (iii) consumption expenditure by industry, E^{ω}_{it}, is given by apparent consumption, $E^{\omega}_{it} = AC^{\omega}_{it} \equiv R^{\omega}_{it} - X^{\omega}_{it} + M^{\omega}_{it}$. Finally, from these figures I construct all the shares relevant to the estimation and quantitative analysis directly from their definitions: (a) trade shares $\lambda^{\omega}_{ij} = X^{\omega}_{ij}/E^{\omega}_{jt}$ for $i \neq j$ and $\lambda_{ii} = 1 - \sum_{j \neq i} \lambda^{\omega}_{ij}$; (b) market shares $\beta^{R,\omega}_{i} = R^{\omega}_{i} / \sum_{j=1}^{N} R^{\omega}_{j}$ and $\beta^{R}_{i} = \sum_{\omega=1}^{\Omega} R^{\omega}_{i} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} R^{\omega}_{j}$; (c) countries' expenditure shares $\beta^{E,\omega}_{i} = E^{\omega}_{i} / \sum_{j=1}^{N} E^{\omega}_{j}$ and $\beta^{E}_{i} = \sum_{\omega=1}^{\Omega} E^{\omega}_{i} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} E^{\omega}_{j}$; (d) Cobb-Douglas parameters $\alpha^{\omega}_{i} = E^{\omega}_{it} / \sum_{\omega=1}^{\Omega} E^{\omega}_{i}$; (v) world-wide industry expenditure shares $\alpha^{\omega} = \sum_{j=1}^{N} E^{\omega}_{j} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} E^{\omega}_{j}$.

[36] Although these two alternative approaches yield the same results in the present model, this is not the case in general. See Melitz and Redding (2015) for a detailed discussion of these two alternative approaches.

B. Estimation Strategy

Taking logs in equation (24) yields the following *comparative advantage equation (CAE)*

$$\text{(CAE)} \qquad \underbrace{\ln\left(\frac{T_{it}^{\omega}/T_{i't}^{\omega}}{T_{it}^{\omega'}/T_{i't}^{\omega'}}\right)}_{\text{Estimable}} = \upsilon \times \underbrace{\ln\left(\frac{\beta_i^{R,\omega}/\beta_{i'}^{R,\omega}}{\beta_i^{R,\omega'}/\beta_{i'}^{R,\omega'}}\right)}_{\text{Observable}} + \underbrace{\ln\left(\frac{\iota_i^{\omega}/\iota_{i'}^{\omega}}{\iota_i^{\omega'}/\iota_{i'}^{\omega'}}\right)}_{\text{Unobservable}}.$$

In this equation, the only unobservable term corresponds to the log of the comparative advantage in innovation, since figures for CAP and market shares can be obtained from production and trade data. Specifically, I construct these figures as follows: (i) I obtain markets shares $\beta_i^{R,\omega}$ from production data as shown in subsection III.A; and (ii) I follow Costinot et al. (2012) and estimate CAP from trade flows according to a procedure that is consistent with the gravity structure of the model as reflected by equation (23.1).[37] Then, (CAE) is an estimable equation that can be taken to the data to get an estimate of parameter υ.

The estimation of the last equation presents some challenges. Per Lemma 1, estimating (CAE) by OLS, treating the unobservable term as an error, yields an inconsistent estimator for υ. In autarky, the endogenous component of CAP is completely determined by relative domestic expenditure, while in a frictionless world it is completely determined by comparative advantage in innovation. Consequently, in any trading equilibrium that is in between these two extreme cases, the endogenous component of CAP should be positively correlated with both, relative domestic expenditure and comparative advantage in innovation. For this reason, we should expect relative market shares to be positively correlated with the unobservable term in (CAE), implying that the OLS estimator of υ is biased upwards.

To address this endogeneity problem I propose to instrument relative market shares in (CAE) with the corresponding double ratios of industry preference parameters under the following assumption.

Assumption A0. *Double ratios of industry preference parameters are uncorrelated with comparative advantage in innovation.*[38]

The rationale of this estimation strategy is based on three observations. (i) There is ample evidence of high trade frictions.[39] (ii) As discussed above, relative market shares are positively correlated with relative domestic expenditure across industries in the presence of high trade frictions. (iii) Relative domestic expenditure is driven, in part, by under-

[37] The details of the estimation procedure can be found in Appendix A.3.1.

[38] As primitive elements of a model, preference parameters and innovation productivities are meant to capture, respectively, intrinsic tastes and innovation abilities that are exogenous to the model. Assumption A0 captures the idea that these intrinsic elements are most likely independent, e.g. the pleasure that a consumer gets from driving a car is likely unrelated to his intrinsic ability to innovate in the automobile industry.

[39] See Anderson and van Wincoop (2004) for a discussion of the empirical literature on trade costs. See Eaton and Kortum (2002) and Waugh (2010) for estimates of trade frictions in Ricardian frameworks.

lying relative industry preference parameters.[40] Observations (i)-(iii) imply that we can instrument relative market shares in (CAE) with double ratios of industry preference parameters under assumption A0.

Before moving to the results, I introduce some simplifying notation and a transformation of the data that I use in the sequel. To avoid potential concerns related to the particular choice of the country and industry relative to which comparative advantage is defined, double ratios of all variables are defined relative to an "average industry" $\bar{\omega}$ and an "average country" $\bar{k}$. The values of variable X that correspond to country k and industry $\bar{\omega}$, $X_{kt}^{\bar{\omega}}$, and to country $\bar{k}$ and industry ω, $X_{\bar{k}t}^{\omega}$, are given by the following geometric averages,

$$X_{kt}^{\bar{\omega}} \equiv \prod_{\omega=1}^{\Omega} \left(X_{kt}^{\omega}\right)^{\frac{1}{\Omega}}, \qquad X_{\bar{k}t}^{\omega} \equiv \prod_{k=1}^{N} \left(X_{kt}^{\omega}\right)^{\frac{1}{N}}. \tag{31}$$

Finally, for any variable X, I use $\bar{X}_{kt}^{\omega}$ to denote the log of double ratios defined relative to the "average industry" $\bar{\omega}$ and the "average country" $\bar{k}$,

$$\bar{X}_{kt}^{\omega} \equiv \ln \frac{X_{kt}^{\omega}/X_{\bar{k}t}^{\omega}}{X_{kt}^{\bar{\omega}}/X_{\bar{k}t}^{\bar{\omega}}}, \tag{32}$$

so that the estimable equation (CAE) can be expressed as $\bar{T}_i^{\omega} = \upsilon \bar{\beta}_i^{R,\omega} + \bar{\iota}_i^{\omega}$.

C. *Estimation and Results*

For the baseline case of Cobb-Douglas preferences, I follow the estimation strategy above and instrument *relative* market shares $\bar{\beta}_i^{R,\omega}$ in (CAE) with the corresponding *relative* industry preference parameters $\bar{\alpha}_i^{\omega}$.[41] Computing $\bar{\alpha}_i^{\omega}$ as relative domestic expenditure across industries $\bar{E}_i^{\omega}$, I obtain the following consistent method of moments estimator of υ,

$$\widehat{\upsilon}_1 \equiv \frac{\sum_{i,\omega} \bar{T}_i^{\omega} \bar{\alpha}_i^{\omega}}{\sum_{i,\omega} \bar{\beta}_i^{R,\omega} \bar{\alpha}_i^{\omega}} = \frac{\sum_{i,\omega} \bar{T}_i^{\omega} \bar{E}_i^{\omega}}{\sum_{i,\omega} \bar{\beta}_i^{R,\omega} \bar{E}_i^{\omega}}. \tag{33}$$

The first column of Table 1 shows the results of naively estimating (CAE) by OLS. The OLS estimator yields a value $\widehat{\upsilon}_{OLS} = 1.023$, slightly above the upper limit of 1 imposed by the theory, although not statistically different from it. However, according to the theory, this estimator should be biased upwards. The second column of the table shows the results obtained using the method of moments estimator $\widehat{\upsilon}_1$. As expected, the estimated

[40]In the case of Cobb-Douglas preferences, relative domestic expenditure is completely determined by double ratios of industry preference parameters as we can see in (25). However, in more general cases relative demand is also affected by prices and income (general CES preferences and homothetic preferences).

[41]When the context is clear, I use the word *relative* when referring to the log-double ratios defined in (32), so that $\bar{\beta}_i^{R,\omega}$ are relative market shares, $\bar{\alpha}_i^{\omega}$ are relative industry preference parameters, etc.

value of υ goes down, with a point estimate of $\widehat{\upsilon}_1 = 0.811$ and the corresponding 95% confidence interval included in the (0, 1) range predicted by the theory.

TABLE 1—ESTIMATION OF υ

D.V.	OLS	IV	IV
log-Comp. Adv. in Prod.		(Expend. Shares)	(Residuals)
	(1)	(2)	(3)
log-Market Shares	1.023	0.811	0.706
	(0.0386)	(0.0472)	(0.0506)
Observations	540	540	540
R-squared	0.566	0.541	0.512

Standard errors in parentheses.

Note: The table shows the results of estimating equation (CAE) according to the three different methods discussed above. Columns (1), (2), and (3) show, respectively, the estimation results corresponding to the OLS estimator, and the method of moments estimators $\widehat{\upsilon}_1$ and $\widehat{\upsilon}_2$. An observation corresponds to a country-industry pair, since double ratios are taken with respect to the (geometric) "average" country and industry defined in the text.

Although most of the quantitative literature assumes the elasticity of substitution across industries to be one at this level of aggregation, $\sigma = 1$, there is little empirical evidence supporting this assumption.[42] In this context, a concern with the estimator in (33) is that the underlying identifying assumption, $\mathbb{E}\left[\bar{E}_i^{\omega}\bar{\iota}_i^{\omega}\right] = 0$, is not expected to hold if elasticity σ differs from one.[43] In particular, if σ exceeds one, then $\mathbb{E}\left[\bar{E}_i^{\omega}\bar{\iota}_i^{\omega}\right] = 0 > 0$ and $\widehat{\upsilon}_1$ is biased upwards, leading to an overestimation of the importance of the endogenous adjustment in technology. Intuitively, a higher relative research productivity leads to a higher relative production technology, a lower relative price and a higher relative domestic expenditure. Below I formalize this argument and propose a solution to address these overestimation concerns.

Suppose the consumption aggregator across industries takes the following CES form

$$C_{it} = \left[\int_{\Omega} \left(\gamma_i^{\omega}\right)^{\frac{1}{\sigma}} C_{it}^{\omega\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{34}$$

where γ_i^{ω} are exogenous industry preference parameters and $\sigma > 1$ is the elasticity of substitution across industries. Also, assumption A0 holds for these preferences, i.e. $\mathbb{E}\left[\bar{\gamma}_i^{\omega}\bar{\iota}_i^{\omega}\right] = 0$. The demand functions corresponding to preferences (34), together with equation (13) that relates price indices P_{it}^{ω} to cost parameters Φ_{it}^{ω}, yield the following log-linear *demand equation,*

$$\bar{E}_i^{\omega} = \tfrac{\sigma-1}{\theta} \times \bar{\Phi}_i^{\omega} + \bar{\gamma}_i^{\omega}. \tag{DE}$$

[42] See Costinot and Rodriguez-Clare (2014) for a discussion of upper-level elasticity assumptions in the quantitative trade literature.

[43] Early studies in the empirical home-market effect literature are subject to a similar concern, an issue that was already raised in Head and Ries (2001), and echoed in Hanson and Xiang (2004) and more recently in Costinot et al. (2019).

Given these new assumptions, the presence of high trade frictions implies that the estimator $\widehat{\upsilon}_1$ in (33), derived under the incorrect Cobb-Douglas assumption, is biased upwards. This can be seen with the help of equations (CAE) and (DE). First, equation (CAE) implies that the log of comparative advantage in innovation $\bar{\iota}_i^{\omega}$ is positively correlated with log of comparative advantage in production $\bar{T}_i^{\omega}$. Second, equation (12) shows that in the presence of high trade costs, the main determinant of the cost parameter Φ_{it}^{ω} is the level of domestic manufacturing technology T_{it}^{ω}.[44] This implies that $\bar{\Phi}_i^{\omega}$ and $\bar{T}_i^{\omega}$ are positively correlated in equilibrium, an implication that is strongly confirmed by the data as we can see from figure 1. Finally, equation (DE) and $\sigma > 1$ imply that $\bar{\Phi}_i^{\omega}$ is positively correlated with relative domestic expenditure $\bar{E}_i^{\omega}$. This sequence of correlations imply that $\bar{E}_i^{\omega}$ and $\bar{\iota}_i^{\omega}$ are positively correlated, violating the identifying assumption above. Moreover, since $\bar{E}_i^{\omega}$ and $\bar{\beta}_i^{R,\omega}$ are positively correlated, it is readily seen that the estimator $\widehat{\upsilon}_1$ is biased upwards.[45] This discussion is summarized in the next Lemma.

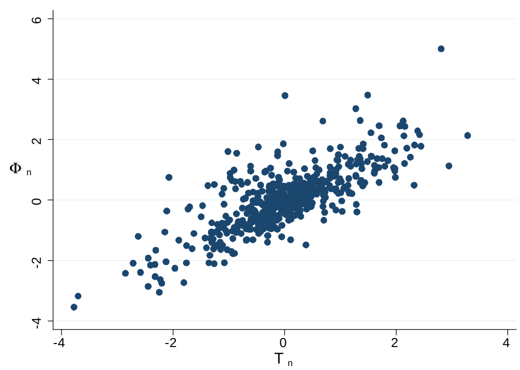

FIGURE 1. RELATIONSHIP BETWEEN $\overline{\Phi}_n$ AND $\overline{T}_n$

LEMMA 3: *Suppose that preferences are given by (34) with $\sigma > 1$, and that they satisfy assumption A0, i.e. $\mathbb{E}\left[\bar{\gamma}_i^{\omega}\bar{\iota}_i^{\omega}\right] = 0$. Then the presence of high trade frictions implies that the method of moments estimator $\widehat{\upsilon}_1$ in (33) is biased upwards.*

Proof: See appendix A.3.2.

It should be clear that if we assume specific values for σ and θ, then we could consistently estimate υ following the same general estimation strategy above. Specifically,

[44]The positive correlation between the Φ_{it}^{ω} and T_{it}^{ω} is an implication of the gravity structure of the model in the presence of high trade frictions, and holds for all values of σ and υ.

[45]The positive correlation between $\bar{E}_i^{\omega}$ and $\bar{\beta}_i^{R,\omega}$ is another implication of the model when trade costs are high, as firms sell a large part of their production domestically.

armed with values for these parameters and figures for $\bar{E}_i^{\omega}$ and $\bar{\Phi}_i^{\omega}$, we could use equation (DE) to back out the relative industry preference parameters $\bar{\gamma}_i^{\omega}$, and use the latter as instruments for relative market shares $\bar{\beta}_i^{R,\omega}$ in equation (CAE). However, given the uncertainty about the value of σ at this level of aggregation, I address the overestimation concerns raised in Lemma 3 following a more robust estimation strategy that does not require assuming specific values for σ and θ. In particular, I complement the upward-biased estimator $\widehat{\upsilon}_1$ with a second estimator, $\widehat{\upsilon}_2$, that is biased downwards if $\sigma > 1$, so that together they provide upper and lower bounds for υ.[46]

Concretely, I construct estimator $\widehat{\upsilon}_2$ as follows. First, I compute $\bar{\Phi}_i^{\omega}$ from equation (11) using estimates of CAP and home trade shares, $\bar{\Phi}_i^{\omega} = \bar{T}_i^{\omega} - \bar{\lambda}_i^{\omega}$. Second, I estimate equation (DE) by OLS treating $\bar{\gamma}_i^{\omega}$ as the error term and compute the OLS residuals, $\widehat{\bar{\gamma}}_i^{\omega}$. Finally, inspired by the general estimation strategy above, I define estimator $\widehat{\upsilon}_2$ as follows,

$$\widehat{\upsilon}_2 = \frac{\sum_{i,\omega} \bar{T}_i^{\omega} \widehat{\bar{\gamma}}_i^{\omega}}{\sum_{i,\omega} \bar{\beta}_i^{R,\omega} \widehat{\bar{\gamma}}_i^{\omega}}. \tag{35}$$

The bias in estimator $\widehat{\upsilon}_2$ reflects a bias in the estimation of $(\sigma - 1)/\theta$ in the first-step regression, which in turn affects the construction of residuals $\widehat{\bar{\gamma}}_i^{\omega}$. Note that if $\upsilon > 0$ and $\sigma > 1$, then equations (CAE) and (DE) imply positive equilibrium correlations between $\bar{\beta}_i^{R,\omega}$ and $\bar{T}_i^{\omega}$, and between $\bar{\gamma}_i^{\omega}$ and $\bar{E}_i^{\omega}$. In addition, from the discussion preceding Lemma 3, high trade frictions imply positive equilibrium correlations between $\bar{E}_i^{\omega}$ and $\bar{\beta}_i^{R,\omega}$, and between $\bar{\Phi}_i^{\omega}$ and $\bar{T}_i^{\omega}$. Taken together, these correlations imply that $\bar{\Phi}_i^{\omega}$ and $\bar{\gamma}_i^{\omega}$ are positively correlated, so the OLS estimator of $(\sigma - 1)/\theta$ in equation (DE) is biased upwards. In turn, this bias in the first-step regression induces a downward bias in $\widehat{\upsilon}_2$. Recall that the right instrument for relative market shares $\bar{\beta}_i^{R,\omega}$ in (CAE) are the industry preference parameters $\bar{\gamma}_i^{\omega}$, which, according to (DE), can be obtained by *adjusting* relative expenditures $\bar{E}_i^{\omega}$ for the effect of prices, $\bar{\gamma}_i^{\omega} = \bar{E}_i^{\omega} - \frac{(\sigma-1)}{\theta}\bar{\Phi}_i^{\omega}$. However, from (33) and (35) we can see that the implicit "instruments" leading to $\widehat{\upsilon}_1$ and $\widehat{\upsilon}_2$ are, respectively, $\bar{E}_i^{\omega}$ and $\widehat{\bar{\gamma}}_i^{\omega} = \bar{E}_i^{\omega} - \widehat{c}\,\bar{\Phi}_i^{\omega}$, where $\widehat{c}$ is the upward-biased estimator of $(\sigma-1)/\theta$. Thus, relative expenditures $\bar{E}_i^{\omega}$ are not adjusted at all for the effect of prices in the first case while are over adjusted in the second case. As a result, the biases in estimators $\widehat{\upsilon}_1$ and $\widehat{\upsilon}_2$ run in opposite directions. These results are stated in the next Lemma.

LEMMA 4: *Suppose that preferences are given by (34) with $\sigma > 1$ and that they satisfy assumption A0, i.e. $\mathbb{E}\left[\bar{\gamma}_i^{\omega}\bar{\iota}_i^{\omega}\right] = 0$. If trade frictions are high and $\upsilon > 0$, then the OLS estimator of $(\sigma - 1)/\theta$ in equation (DE) is biased upwards and the method of moments estimator $\widehat{\upsilon}_2$ in (33) is biased downwards.*

Proof: See Appendix A.3.3.

[46]Both estimators provide lower bounds for υ when $\sigma < 1$, underestimating the mechanism proposed in this paper.

The second and third columns of Table 1 show the results obtained using estimators $\widehat{\upsilon}_1$ and $\widehat{\upsilon}_2$, respectively. As we can see, the point estimates corresponding to these estimators—$\widehat{\upsilon}_1 = 0.811$ and $\widehat{\upsilon}_2 = 0.706$—satisfy the inequality implied by Lemmas 3 and 4, so together they provide upper and lower bounds for υ under the assumption $\sigma \geq 1$.[47] These results suggest that there is scope for the effects of directed research to be quantitatively important as even the estimated lower bound for υ exceeds 0.5, the threshold value given in Lemma 2 above which the endogenous adjustments in technology are strong enough to allow for the possibility of home market effects and potential reversals in the export profile of countries.[48]

IV. Quantitative Analysis

In this section I explore the quantitative relevance of the effects of directed research along two dimensions. First, I asses the importance of the endogenous adjustments in technology in the determination of CAP and explore how this process is affected by trade. Second, I investigate how the new margin of adjustment affects the answer to some of the standard questions in the quantitative trade literature, including the effect of trade costs on production, trade flows and real income.

A. Calibration

In the baseline calibration of the model I set $\upsilon = 0.758$, corresponding to the midpoint of the interval delimited by the estimates $\widehat{\upsilon}_2 = 0.706$ and $\widehat{\upsilon}_1 = 0.811$ obtained in section III. As a robustness check, I also present results for values of υ corresponding to the endpoints of that interval.[49] Finally, I set the shape parameter θ to 4, which is within the range of proposed values for this parameter in the literature.[50]

B. Results

ENDOGENOUS COMPARATIVE ADVANTAGE AND TRADE. — Armed with estimates of CAP, figures for market shares and a value for υ, equations (24) and (CAE) can be used to decompose the log of CAP, $\bar{T}_i^{\omega}$, as the sum of the endogenous component $\upsilon \bar{\beta}_i^{R,\omega}$ and the

[47]In a recent working paper, Costinot et al. (2019) estimate the supply elasticity in the pharmaceutical industry and obtain results that are consistent with the estimates of υ above. Combining equations (23.1), (23.4) and (23.8) yields $\lambda_{ij}^{\omega} = \delta_i^{\omega} + \psi_i^{\omega} + \xi_{ij} + \upsilon \ln \beta_i^{R,\omega} + \varepsilon_{ij}^{\omega}$, where ξ_i^{ω}, ψ_j^{ω}, ξ_{ij} are exporter-industry, importer-industry and importer-exporter fixed effects. This is essentially the same equation that Costinot et al. (2019) estimate following an intrumental variable approach. Their estimate of the coefficient on the sales variable, corresponding to υ in this context, is 0.779 and falls within the bounds estimated above.

[48]In appendix A.3.4, I explore biases in the estimates of υ that may arise if consumer's preferences are non-homothetic, and conclude they are not quantitatively important in the context of this paper.

[49]I keep the Cobb-Douglas assumption for preferences throughout the analysis to isolate the quantitative effects of alternative assumptions about the value of υ.

[50]Costinot, Donaldson and Komunjer (2012) obtain a value $\theta = 6.53$ using a static multi-industry model in which θ is common across industries. However, their IV estimation procedure yields an upward biased estimator in the present model due to the two-way relationship between trade flows and R&D that arises as a consequence of directed research.

exogenous component $\bar{\iota}_i^{\omega} = \bar{T}_i^{\omega} - \upsilon\bar{\beta}_i^{R,\omega}$, capturing the log of comparative advantage in innovation. Then, a measure of the quantitative importance of the endogenous adjustments in technology is given by the contribution of the endogenous component $\upsilon\bar{\beta}_i^{R,\omega}$ to the total variance of $\bar{T}_i^{\omega}$.[51]

The second row of table 2 shows the results of this variance decomposition corresponding to the observed open equilibrium in 2006. For the baseline value of $\upsilon = 0.758$, the share of the endogenous component in the total variance of CAP is 52.8%, indicating that the endogenous adjustments in technology play an important role in shaping CAP. Also, the share of the endogenous component is robust with respect to the value of the parameter υ in the range delimited by the lower and upper bounds estimated in the last section: 51.2% for $\upsilon = 0.706$ and 54.2% for $\upsilon = 0.811$.

TABLE 2—ENDOGENOUS COMPONENT OF COMPARATIVE ADVANTAGE IN PRODUCTION.

	$\upsilon = 0.706$	$\upsilon = 0.758$	$\upsilon = 0.811$
	%	%	%
Zero Gravity	91.4	94.1	96.4
Actual Open Equilibrium	51.2	52.8	54.2
Autarky	25.7	26.2	26.3

Note: The table shows the share of the endogenous component in the total variance of log-comparative advantage in production for different values υ and for alternative assumptions about the structure of trade frictions. The selected values for the decreasing returns parameter υ include the estimated lower and upper bounds estimated in the text and the benchmark calibration for υ corresponding to the average of the bounds.

How important are the adjustments in technology *induced by trade*? To answer this question I analyze how the variance decomposition above is affected by trade costs. Armed with the figures for the log of comparative advantage in innovation obtained in the decomposition above and values for the Cobb-Douglas preference parameters, $\bar{\alpha}_i^{\omega} = \bar{E}_i^{\omega}$, I use Lemma 1 to obtain a similar variance decomposition of the log of CAP, $\bar{T}_i^{\omega}$, for the cases of frictionless trade and autarky. The results are shown in the first and third rows of Table 2. For the baseline value of $\upsilon = 0.758$, moving from autarky to the observed equilibrium in 2006 increases the share of the endogenous component from 26.2% to 52.8%, implying that trade is responsible for about a fourth of the total variance of $\bar{T}_i^{\omega}$ in the observed equilibrium in 2006. In addition, the share of the endogenous component increases to 94.1% in the frictionless trade equilibrium, suggesting that there is plenty of room for further adjustments in technology as trade frictions decline. These results show that trade has a significant impact on technology.

The estimated endogenous component of CAP indicates that the observed equilibrium in 2006 is characterized by high trade frictions. For the baseline value of $\upsilon = 0.758$, panel A of figure 2 shows a tight connection between the endogenous components in the observed equilibrium and in autarky. In contrast, panel B of the figure shows a much

[51]This variance decomposition is not intended to assess the relative contributions of exogenous elements of the model in the determination of comparative advantage in production. Instead, it is intended to assess the importance of the endogenous adjustments of technology induced by directed research, whatever the determinants of those adjustments are.

weaker correlation between the endogenous components of comparative in the observed equilibrium and in the zero gravity world.[52]

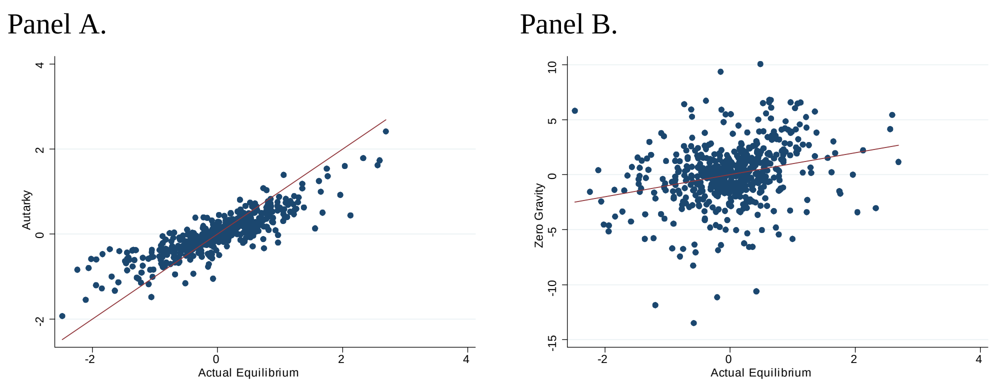

FIGURE 2. TRADE FRICTIONS AND THE ENDOGENOUS COMPONENT OF CAP

Note: The relationships depicted correspond to the innovation model with $\upsilon = 0.758$. In red, the 45 degree line.

MOVING TO AUTARKY. — For this counterfactual scenario I focus on the effects on real income, as the effects on trade flows are trivial. As explained in section II.C, the reductions in the BGP-levels of real income per-capita as countries move to autarky are computed according to appropriate version of (30),

$$\ln \frac{W_{it}^{a}}{W_{it}} = \frac{\upsilon}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\alpha_i^{\omega}}{\delta_i^{\omega}} \right) d\omega + \frac{1}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\lambda_{ii}^{\omega} \right) d\omega, \tag{36}$$

where W_{it}^{a} and W_{it} denote the real income in autarky and in the actual equilibrium, respectively. Taking 2006 as the initial equilibrium, table 3 shows the decline in real income implied by (36) for a range of values of the R&D parameter υ that includes the upper and lower bounds estimated previously.[53] Column 1 shows the predictions of the model with no innovation, $\upsilon = 0$, corresponding to the second term in (36). The other columns show the changes in real income for positive values of υ *relative* to the changes corresponding to the no innovation model in column 1,

$$\frac{\ln W_{it}^{a,\upsilon} / W_{it}}{\ln W_{it}^{a,0} / W_{it}} = 1 + \upsilon \frac{\int_{\Omega} \alpha_i^{\omega} \ln \left(\alpha_i^{\omega} / \delta_i^{\omega} \right) d\omega}{\int_{\Omega} \alpha_i^{\omega} \ln \left(\lambda_{ii}^{\omega} \right) d\omega}.$$

[52] Note that table 2 gives a similar message, as the share of the endogenous component in the observed equilibrium in 2006 is closer to the corresponding autarky share than to the zero gravity share.

[53] For all counterfactuals, trade deficits are first closed according to the innovation model with $\upsilon = 0.758$.

Per proposition 2.ii, the model with no innovation overestimates the reduction in real income from moving to autarky, so the first term in (36) and the second term in the last expression are necessarily positive and negative, respectively. Accordingly, the values in columns 2-4 are strictly below one, with higher values of υ associated with lower reductions in real income. However, the general picture emerging from Table 3 is that the relative differences in the predicted changes in real income between the models with and without innovation appear to be quantitatively modest. For the case of the benchmark value of $\upsilon = 0.758$, the reductions in real income relative to the model with no innovation range from 72% for Australia, to 98% for Belgium-Luxembourg, with a mean value for the sample of 93%.

These modest quantitative differences between the models reflect two features of the data that affect the *relative* importance of the first term in (36), the only source of these differences for this counterfactual. First, most countries exhibit small differences between their production specialization profiles in the actual open equilibrium and in autarky, as captured by δ_i^{ω} and α_i^{ω}, respectively.[54] Figure 3 shows a close connection between α_i^{ω} and δ_i^{ω} for the year 2006, with a correlation coefficient of 0.81. Accordingly, for the average country, the first term in (36) is small in absolute value. And second, the first term in (36) tends to be important for those countries in which the second term is also important (in absolute terms), with a correlation of -0.76 between the two terms for the baseline value of $\upsilon = 0.758$. Per these two factors, the second term of the last expression is small in absolute value for most countries.

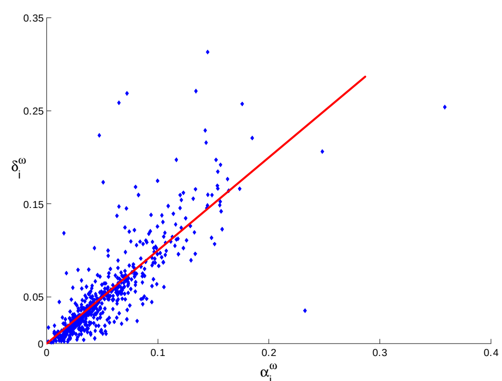

FIGURE 3. ALLOCATION OF RESOURCES. ACTUAL EQUILIBRIUM VS. AUTARKY.

Note: The dots in the figure correspond to country-industry pairs. In red, the 45 degree line.

UNIFORM 25% REDUCTION IN TRADE COSTS ACROSS INDUSTRIES AND COUNTRIES. — As discussed above, all changes in equilibrium variables considered in this counterfactual

[54]In autarky, the share of industry ω in total output is completely driven by domestic demand, $\delta_i^{a,\omega} = \alpha_i^{\omega}$.

are computed using the system (23) in changes (system A.8 in the appendix), calibrated to the observed equilibrium in 2006. I start by analyzing the effects of directed research on trade flows. Panel A of figure 4 shows the relationship between the log-changes in trade shares predicted by the baseline innovation model ($\upsilon = 0.758$) and by the model with no innovation, together with the 45 degree line in red. Regressing the former on the latter yields a slope coefficient of 1.05, implying that, on average, the direction and magnitude of the predicted changes in trade shares are similar in both models. However, as we can see from the figure, this average hides significant differences in the predictions of both models. The R-square of said regression is 0.378, i.e., only a little more than a third of the variation in the trade flow changes predicted by the model with directed research can be explained by those predicted by the model with no innovation. In addition, in about a fourth of the cases, the predicted changes in trade shares in both models go in opposite directions.

Panel B of figure 4 shows a scatter plot of the log-changes in market shares $\beta_i^{R,\omega}$ predicted by the models with and without innovation, together with the 45 degree line in red. As we can see, the direction of these changes is similar in both models, with a correlation coefficient of 0.94 between the two variables. However, the model with no innovation underestimates the magnitude of the responses in market shares relative to the model with directed research. A regression of the predicted log-changes in market shares in the model with directed research on its counterpart in the model with no innovation yield a slope coefficient of 3.37.

Panel A. Log Changes in Trade Shares Panel B. Log Changes in Market Shares

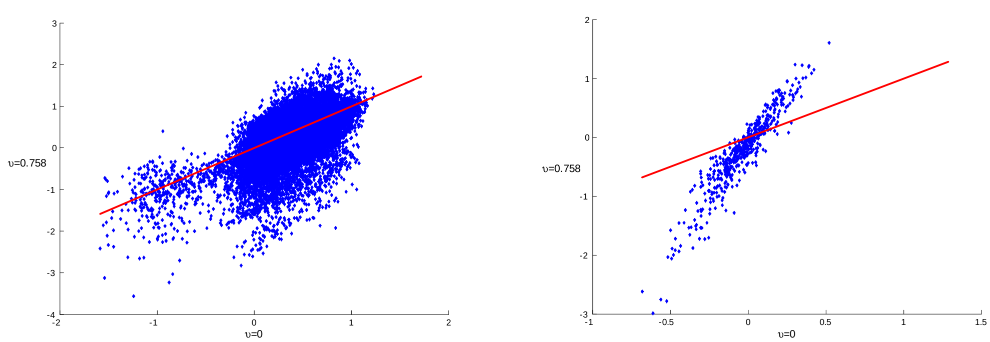

FIGURE 4. DECLINE IN TRADE COSTS: INNOVATION VS. DIRECTED RESEARCH

Note: Each dot in panel A represents an exporter-importer-industry triplet, $\log \widehat{\lambda}_{ij}^{\omega}$, while those in panel B represent country-industry pairs, $\log \widehat{\beta}_i^{R,\omega}$. In red, the 45 degree line.

Table 4 shows the implied changes in real income per capita for the same values of the R&D parameter υ considered in the previous exercise. These changes can be computed as

$$\ln \frac{W'_{it}}{W_{it}} = \frac{\upsilon}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\delta_i'^{\omega}}{\delta_i^{\omega}} \right) d\omega - \frac{1}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\lambda_{ii}'^{\omega}}{\lambda_{ii}^{\omega}} \right) d\omega, \tag{37}$$

where W'_{it} denotes the real income in the new counterfactual equilibrium and W_{it} denotes its counterpart in the initial open equilibrium in 2006. As in table 3, Columns (2)-(4) present the predicted changes in real income relative to those obtained in the no-innovation model ($\upsilon = 0$), shown in column 1. Three messages arise from the table. First, real income rises in all countries for all values of the parameter υ. Second, as suggested by proposition 2.i, the model with no innovation tends to underestimate the increases in real income for most countries (only 6 exceptions). Third, the predictions of the models with and without innovation are quantitatively similar. For the baseline innovation model with $\upsilon = 0.758$, the predicted changes in real income relative to those of the model with no innovation range from 83% for China, to 108% for Japan, with a mean value for the sample of 102%.

The last result may be surprising to some readers, as the previous results show that directed research significantly affect the change in trade flows entering the second term in (37). Indeed, directed research has a significant impact in both terms in (37), as shown in table 5. The first and second columns of the table show, respectively, the predicted change in real income corresponding to the model with no innovation and to the baseline innovation model with $\upsilon = 0.758$. The third and fourth columns decompose the latter into the second (home shares) and first (industry shares) terms in (37). As the first term of (37) is zero for the model with no innovation, the difference between columns 3 and 1 captures the impact of directed research on the second term in (37), while column 4 reflects its impact on the first. Although the effect of directed research on each term is significant, these effects work in opposite directions, resulting in a modest overall effect on the predicted change in real income.

The intuition behind this result is simple. Let X'' and X' denote the values of variable X after the 25% reduction in trade costs in the model with directed research ($\upsilon > 0$) and in the model with no innovation, respectively. After the shock, country i's industry price levels in industry ω in the two models satisfy

$$\frac{P_{it}''^{\omega}}{P_{it}'^{\omega}} \propto \left(\Phi_{it}''^{\omega} / \Phi_{it}'^{\omega} \right)^{-\frac{1}{\theta}} = \left(\frac{\lambda_{it}''^{\omega} / \lambda_{it}'^{\omega}}{T_{it}''^{\omega} / T_{it}'^{\omega}} \right)^{\frac{1}{\theta}}, \tag{38}$$

where the differences between X'' and X' reflect the endogenous adjustments in technology allowed by directed research. Suppose that in the innovation model, country i innovates more in industry ω following the reduction in trade costs, inducing a rise in manufacturing technology from $T_{it}'^{\omega}$ to $T_{it}''^{\omega}$.[55] The same specialization process induces other countries to reduce their innovation in that industry. As a result, country i's home

[55] $T_{it}'^{\omega}$ also represents the level of technology in the innovation model before the shock.

share of expenditure in the industry is higher in the innovation model relative to the model with no innovation, $\lambda_{it}^{\prime\omega} < \lambda_{ii}^{\prime\prime\omega}$. A symmetric analysis shows that manufacturing technology and home shares of expenditure also move in the same direction when innovation reallocates away from the industry. As a result, the effects of directed research on manufacturing technology and on the home share of expenditure in (38) work in opposite directions, leading to overall modest effects on industry price indices and real income.

EXTENSIONS OF THE MODEL. — In appendix B I extend the baseline model to include multiple factors of production, heterogeneous trade elasticities across industries and intermediate inputs, and discuss how these extensions may affect the quantitative results above. I argue that many of these results seem to be more general than what the simple structure of the baseline model would suggest, as some of these extensions just affect the interpretation of some elements of the model. Accounting for multiple factors of production and heterogenous trade elasticities has little impact on the estimated value of the R&D parameter υ and on our conclusions regarding the *relative* importance of directed research in the determination of CAP and for welfare evaluations. Interestingly, including intermediate goods reduces the estimated value of υ, but has little impact on many of the other results.[56] This is the case because the presence of intermediate goods tends to amplify the overall effect of directed research on CAP for a given value of υ, and this overall effect is what the estimations in the baseline model are capturing. In all cases, the main messages of the paper go through, i.e. directed research is an important determinant of CAP and trade flows, but it is a somewhat less important factor to understand the effects of trade in manufactured goods on aggregate real income.

V. Conclusions

In this paper I develop a multi-country, general equilibrium, semi-endogenous growth model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of the agents to direct their research efforts to specific industries, in a context of heterogeneous innovation capabilities and contemporaneous decreasing returns to R&D. As a result, trade can affect the direction of innovation and the distribution of technology in the BGP, so the model features a two-way relationship between trade and technology absent in standard quantitative Ricardian trade models. An attractive feature of the model is that the strength of these endogenous adjustments in technology is controlled by a single parameter, which I estimate using production and trade data from 2006 for a sample of 29 countries and 18 manufacturing industries.

I use the model to disentangle the effects of trade on technology and to study questions that standard Ricardian quantitative trade models are not suitable to answer. Under the baseline calibration of the model, I find that the endogenous adjustments in technology

[56] The estimated value of υ declines to 0.584, but the variance of overall CAP observed in the open equilibrium is 51.5%, only slightly below the 52.8% obtained in the baseline model.

due to directed research can account for up to 52.8% of the observed variance in comparative advantage in production in the observed trading equilibrium in 2006. I also show that allowing for endogenous adjustments in technology induces significant differences in the adjustments of trade flows and market shares in response to a 25% reduction in trade costs. Finally, I show that the standard Ricardian model with no innovation overestimates the reduction in real income from moving to autarky and tends to underestimate the increases in real income from reductions in trade costs. However, notwithstanding the relevant effects of directed research on technology, production and trade flows, the predicted changes in real income associated with moving to autarky and with a 25% reduction in trade costs do not differ much across models.

Finally, I show that many of the results obtained in the baseline model are more general than what its simple structure would suggest, as they are little changed after accounting for multiple factors of production, heterogeneous trade elasticities across industries and intermediate inputs. In all of these extensions the main messages of the paper go through, i.e. directed research is an important determinant of comparative advantage in production and trade flows, but it is a somewhat less important factor to understand the effects of trade in manufactured goods on aggregate real income.

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TABLE 3—CHANGES IN REAL INCOME. MOVING TO AUTARKY. MANUFACTURING SECTOR.

	$\upsilon = 0$	$\upsilon = 0.706$	$\upsilon = 0.758$	$\upsilon = 0.811$
	Level %	Rel. to (1)	Rel. to (1)	Rel. to (1)
	(1)	(2)	(3)	(4)
AUS	-16.67	0.74	0.72	0.70
AUT	-36.82	0.98	0.98	0.98
BLX	-41.96	0.99	0.98	0.98
BRA	-6.75	0.90	0.89	0.89
CAN	-30.55	0.96	0.95	0.95
CHE	-26.84	0.93	0.93	0.92
CHN	-12.75	0.83	0.81	0.80
CZE	-26.01	0.95	0.94	0.94
DEU	-20.33	0.98	0.98	0.97
DNK	-30.31	0.92	0.91	0.91
ESP	-14.40	0.97	0.97	0.97
FIN	-21.60	0.94	0.94	0.93
FRA	-15.76	0.98	0.98	0.98
GBR	-17.94	0.98	0.98	0.98
GRC	-28.54	0.85	0.84	0.83
HUN	-37.53	0.98	0.97	0.97
IRL	-30.43	0.87	0.86	0.85
ISR	-33.80	0.85	0.84	0.83
ITA	-13.49	0.91	0.90	0.90
JPN	-6.58	0.96	0.95	0.95
KOR	-11.61	0.97	0.97	0.97
MEX	-28.14	0.96	0.96	0.96
NLD	-31.98	0.97	0.97	0.97
NOR	-20.76	0.92	0.92	0.91
POL	-19.99	0.94	0.94	0.93
PRT	-27.88	0.97	0.96	0.96
SGP	-61.43	0.88	0.88	0.87
SWE	-23.28	0.96	0.96	0.95
USA	-8.24	0.96	0.96	0.95
mean	-24.22	0.93	0.93	0.92
median	-23.28	0.96	0.95	0.95
min	-6.58	0.74	0.72	0.70
max	-61.43	0.99	0.98	0.98

Note: The levels in column (1) are calculated for a value $\theta = 4$. The other columns represent the change in real income relative to column (1); this relative measure is not affected by the value of θ.

TABLE 4—CHANGES IN REAL INCOME. 25% REDUCTION IN TRADE COSTS.

	$\upsilon = 0$	$\upsilon = 0.706$	$\upsilon = 0.758$	$\upsilon = 0.811$
	Level %	Rel. to (1)	Rel. to (1)	Rel. to (1)
	(1)	(2)	(3)	(4)
AUS	13.71	1.02	1.03	1.05
AUT	24.31	1.02	1.03	1.04
BLX	25.45	1.04	1.05	1.07
BRA	6.32	1.00	1.00	1.00
CAN	27.35	1.01	1.01	1.02
CHE	20.01	0.99	0.98	0.98
CHN	8.23	0.86	0.83	0.79
CZE	19.19	1.01	1.01	1.01
DEU	15.34	1.03	1.04	1.05
DNK	22.03	1.03	1.04	1.06
ESP	12.55	1.01	1.01	1.01
FIN	15.98	1.02	1.03	1.03
FRA	14.18	1.04	1.05	1.06
GBR	15.60	1.02	1.02	1.03
GRC	16.45	1.01	1.01	1.01
HUN	22.10	0.99	0.99	0.99
IRL	19.81	1.02	1.03	1.04
ISR	19.21	0.98	0.97	0.95
ITA	11.39	1.03	1.04	1.05
JPN	6.44	1.07	1.08	1.11
KOR	10.49	1.05	1.06	1.07
MEX	21.64	0.97	0.97	0.97
NLD	23.64	1.05	1.07	1.09
NOR	16.35	0.99	0.99	0.99
POL	15.60	1.01	1.02	1.02
PRT	19.54	1.02	1.02	1.03
SGP	24.69	0.99	1.00	1.00
SWE	18.59	1.02	1.02	1.02
USA	7.59	1.04	1.05	1.07
mean	17.03	1.01	1.02	1.02
median	16.45	1.02	1.02	1.03
min	6.32	0.86	0.83	0.79
max	27.35	1.07	1.08	1.11

Note: The levels in column (1) are calculated for a value $\theta = 4$. The other columns represent the change in real income relative to the column (1).

TABLE 5—DECOMPOSITION OF CHANGES IN REAL INCOME. 25% REDUCTION IN TRADE COSTS.

	$\upsilon = 0$	$\upsilon = 0.758$		
	Level %	Level %	Home Shares	Industry Shares
	(1)	(2)	(3)	(4)
AUS	13.71	14.13	33.25	-19.12
AUT	24.31	25.07	27.84	-2.76
BLX	25.45	26.75	28.19	-1.44
BRA	6.32	6.32	9.47	-3.15
CAN	27.35	27.68	37.01	-9.34
CHE	20.01	19.65	25.55	-5.89
CHN	8.23	6.85	10.98	-4.13
CZE	19.19	19.36	22.50	-3.14
DEU	15.34	15.94	17.44	-1.50
DNK	22.03	22.99	29.05	-6.06
ESP	12.55	12.67	14.24	-1.58
FIN	15.98	16.42	20.64	-4.22
FRA	14.18	14.82	16.17	-1.34
GBR	15.60	15.93	18.02	-2.10
GRC	16.45	16.56	20.05	-3.49
HUN	22.10	21.91	24.48	-2.57
IRL	19.81	20.34	24.33	-3.99
ISR	19.21	18.58	25.97	-7.39
ITA	11.39	11.83	13.26	-1.42
JPN	6.44	6.99	8.91	-1.92
KOR	10.49	11.13	12.44	-1.31
MEX	21.64	20.96	29.90	-8.93
NLD	23.64	25.25	31.53	-6.29
NOR	16.35	16.14	20.77	-4.63
POL	15.60	15.87	18.82	-2.95
PRT	19.54	19.97	21.22	-1.25
SGP	24.69	24.59	30.25	-5.66
SWE	18.59	18.94	21.98	-3.04
USA	7.59	8.01	10.19	-2.18
mean	17.03	17.30	21.53	-4.23
median	16.45	16.56	21.22	-3.14
min	6.32	6.32	8.91	-19.12
max	27.35	27.68	37.01	-1.25

Note: Columns (1) and (2) show the change in real income associated with a 25% reduction in trade costs for the indicated values of υ. The figures are calculated for a value of $\theta = 4$. Columns (3) and (4) decompose column (2) into the two terms in (37) such that column 2 is the sum of columns 3 and 4.

TABLE 6—LIST OF MANUFACTURING INDUSTRIES.

ISIC Rev. 3	Description
15-16	Manufacture of food products, beverages and tobacco products.
17-19	Manufacturing of textiles, wearing apparel and leather products.
20	Manufacture of wood and wood products (excl, furniture).
21-22	Manufacture of paper products, publishing and printing.
23	Manufacture of coke and refined petroleum products.
24	Manufacture of chemicals and chemical products.
25	Manufacture of rubber and plastic products
26	Manufacture of other non-metallic mineral products.
27	Manufacture of basic metals.
28	Manufacture of fabricated metal products.
29	Manufacture of machinery and equipment.
30	Manufacture of office, accounting and computing machinery.
31	Manufacture of electrical machinery n.e.c.
32	Manufacture of radio and television equipment.
33	Manufacture of medical, precision and optical instruments.
34	Manufacture of motor vehicles and trailers.
35	Manufacture of other transport equipment.
36-37	Manufacture of furniture and recycling.